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AN INVESTIGATION INTO THE ROLE OF DIAGRAMS IN PROBLEM SOLVING IN MATHEMATICS AT FORM FOUR LEVEL AT CHIFEDZA HIGH SCHOOL IN CHIVI DISTRICT.

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DECLARATION

I, Godfree Magwenzi do hereby declare that the work contained in this dissertation is entirely my own work with the exception of such quotations or references which have been attributed to their authors or sources and that I have not previously submitted it at any university for a degree.

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ABSTRACT

This study which was conducted at Chifedza High School in Chivi District of Masvingo Province of Zimbabwe sought through a survey and quasi experimental, to investigate the role of diagrams in problem solving in Mathematics at form four level. The exploratory approach was chosen to guide the methodology of the study because of its pragmatic nature and ability to produce desired, practical, workable and more justifiable results. One secondary school in the district was visited for the study. Teachers, formal and non-formal students were visited for the study. The actual study sample consisted of twenty five formal form four students, five non-formal students and ten mathematics teachers drawn from the Mathematics and Science departments making a total of forty respondents and informants.

Students were subjected to two tests and a questionnaire. Teachers completed a questionnaire. All the questionnaires had closed and open-ended questions to supplement the quantitative data. Data was presented in tables and graphs. Data was analysed through the Z-test for comparison of means. Qualitative data from open ended sections of the questionnaires were analysed by first noting frequencies of each response, categorising the responses and then finding the emerging themes from the responses.

Results of the study revealed that in general students performed better in diagrammatic problems than word problems. In particular, teachers and students appreciated the role of diagrams in problem solving in mathematics. Quantitative results confirmed that students performed better in the second test where diagrams were used. There was low performance in the first test where word problems were used, an indication that diagrams play a pivotal role

when solving problems in mathematics.

The conclusion of the study was that diagrams helped learners to solve problems in mathematics. It was also concluded that teachers should incorporate diagrams in problem solving in mathematics and learner centred approaches are essential in helping learners understand the problem, devising plans and strategies, implementing devised plans and strategies and reviewing and reflecting solutions. It was from these conclusions that recommendations and proposals were made in this study. It is strongly anticipated that if the recommendations are implemented, then the role of diagrams in problem solving in mathematics can be fully realised and appreciated.

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DEDICATION

I dedicate this dissertation firstly to God Almighty. May all those who read it and make use of it give all the credits, praises and honour to him.

Secondly, to my lovely wife Elima, our children Tinotenda and Tatenda whose love, prayers, encouragement and moral support have kept me going.

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TABLE OF CONTENTS

PAGE

Approval Form…….....i

Release Form……...ii

Declaration……..iii

Abstract……...iv

Acknowledgements…….....vi

Dedication……..vii

Table of Contents……..viii

CHAPTER 1: THE PROBLEM AND ITS SETTING

1.0 Introduction……..……1

1.1 Background of the problem……...1

1.2 Statement of the problem……...……...5

1.3 Research Objectives……...……...6

1.4 Research Questions…….....6

1.5 Significant of the study……..……..7

1.6 Delimitations of the study……..…….....9

1.7 Limitations of the study…….....9

1.8 Definition of terms……..…….....10

1.9 Chapter Summary……..…….....11

CHAPTER 2: REVIEW OF RELATED LITERATURE

2.0 Introduction…….……..12

2.1 Theoretical Literature Review…….…….....12

2.1.1 Philosophical Background……..…….……....12

2.1.2 Psychological Background……..…….……...14

2.1.3 George Polya's four step model of problem solving……16

2.2 Conceptual meaning of problem solving…….16

2.3 The role of diagrams in helping learners to show understanding of mathematical problems…….17

2.4 The role of diagrams in helping learners to devise and design plans for solving mathematical

problems……..…….18

2.5 The role of diagrams in helping learners in implementing devised plans and strategies in solving mathematical problems. 19

2.6 The role of diagrams in helping learners to review and verify solutions obtained in solving mathematical

problems.....20

2.7 Chapter Summary……...……20

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.0 Introduction…….…….22

3.1 Research design…….....22

3.2 Population…….…….....23

3.3 Sample and sampling procedures……..…….....24

3.4 Research instruments…….....27

3.4.1 Questionnaires……...27

3.4.2 Tests……..29

3.5 Data collection procedures……..…….30

3.5.1 Sources of data……..31

3.6 Validity, Reliability and Objectivity……32

3.7 Ethical considerations……..33

3.8 Data Presentations and Analysis……..…….....33

3.9 Chapter summary…….34

CHAPTER 4: RESEARCH FINDINGS, DATA PRESENTATION AND DATA ANALYSIS

4.0 Introduction……..35

4.1 Data presentation and analysis…….35

4.1.1 Biographic data…….35

4.2 Responses to research questions……..42

4.2.1 Responses on the role of diagrams in helping learners to show understanding of mathematical problems……..42

4.2.2 Responses on the role of diagrams in helping learners to devise and design plans for solving mathematical problems…….43

4.2.3 Responses on the role of diagrams in helping learners implementing devised plans and strategies in solving mathematical problems…….....44

4.2.4 Responses on the role of diagrams in helping learners review, reflect and verify solutions obtained in solving mathematical problems……..45

4.3 Data discussion and analysis……50

4.3.1 What is the role of diagrams in helping learners show understanding of mathematical problems?……...50

4.3.2 What is the role of diagrams in helping learners to devise and design plans for solving mathematical problems?…….....51

4.3.3 What is the role of diagrams in helping learners implementing devised plans and strategies in solving mathematical problems?.....51

4.3.4 What is the role of diagrams in helping learners to review, reflect and verify solutions obtained in solving problems……..…….52

4.4 Chapter summary……..…….....52

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction……..…….....54

5.1

Summary……...……..54

5.2 Conclusion……...……...55

5.3 Recommendations…….…….……56

References…….....58

Appendices…….....63

CHAPTER ONE: THE PROBLEM AND ITS SETTING

1.0 INTRODUCTION

This chapter provided the background to the problem which highlighted what has really prompted the writer to undertake the investigation. It outlined the problem statement which probed the extent and dimension from which the writer carried out the investigation. It further stated the research questions and objectives to further define the direction on which the research was carried out. The importance of the study winded off the first chapter.

1.1 BACKGROUND TO THE PROBLEM

Mathematics at Ordinary Level is one of the subjects which cause a lot of concern whereby average pupils fail to pass on the first attempt. On the other hand, Mathematics together with English Language and Integrated Science form the backbone and basic requirement for both academic and vocational advancement. Of the three subjects, Mathematics has the lowest pass rate both at national and secondary school level. At Chifedza High School where most of the research work is going to be done the pass rates are also very low. Chifedza High School enrols learners with units ranging from four units at grade seven to thirty six units, that is, learners of mixed abilities. However, the achievement in Mathematics has and is always lagging. Given below is the summary of results at Chifedza High School for the past seven years.

Table 1.1.1: % pass rate for different subjects for the past seven years

Subject/Year	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Mathematics	**20**	**12**	**15**	**11**	**15**	**16**	**11**
English	25	13	17	12	17	20	10
Int. Science	35	40	35	50	61	30	25
Shona	60	45	68	79	89	75	91
Geography	54	76	87	45	45	78	67
History	78	70	87	90	68	79	87
Accounting	34	56	87	46	75	50	54
R/S	60	67	89	95	79	74	89
Commerce	56	67	80	51	50	76	90
Agriculture	78	90	76	56	68	87	80
F/F	45	65	35	70	56	67	50

By nature Mathematics is a subject which cannot be taught or learned using rigorous teaching strategies (Diezmann and English, 2001) but, due to the traditional adage by which rote learning, drill and lecture methods dominate the process of teaching and learning in the name of fast syllabus coverage learners fail to understand and realise the importance of mathematics.

Mathematics is all about problem solving, therefore, anyone who is competent in problem solving does not usually have problems in Mathematics. In solving mathematical problems learners sometimes need to design graphical representations of the problem itself. Cognitive and learning Psychologists Jerome Bruner and Jean Piaget subscribe to the idea that human learning should proceed from iconic and concrete representations to symbolic and abstract formal modes of operation. Towards this end, this investigation was compelled to find the role of diagrams in

problem solving in Mathematics. Diezmann and English (2001) put forward that, a diagram has an advantage in that it can serve as a cognitive tool to display information in a spatial manner which in turn allows the learner to understand the problem and subsequently solve the problem. This investigation therefore, took the task to find out how far 'iconic' and 'concrete' diagrammatical and graphical representation help learners to, understand a mathematical problem, design and apply strategies to solve mathematical problems and represent solutions to mathematical problems. According to Bruner (1960: 257), "any subject matter can be taught successfully to any learner at any level as long as the content is prepared to suit the level of the learner." Undoubtedly, diagrams and graphical presentations of any mathematical problem remove linguistic and computational complexities. In short, diagrams can basically be understood even by the semi-literate learners as they provide a concrete picture of any stage of the problem being solved. As most if not all, learning theories propose that all learning should proceed from concrete to abstract and from simple to complex, diagrams occupy the centre stage in representing any abstract mathematical concept in concrete and simple manner. However, the concrete and simple diagrammatical representations call for some inquiry into how far they can help learners to solve mathematical problems.

According to the Examiners' Report, Mathematics 4008/4028 Paper 2 of 2012, candidates performed badly in word problems and did quite well in problems with diagrammatic representations. It was against this background of high failure rate in mathematics, traditional teaching and learning strategies and iconic and concrete diagrammatic appeal that drew this researcher to investigate the role of diagrams in solving problems in Mathematics. Pantziara, Gagatsis and Pitta-Pantazi (2004) carried out a study to investigate the role of diagrams in solving non-routine problems in Mathematics. The study was carried out in Cyprus. Two tests were used in their study, test A and test B. The tests were administered on 194, twelve year old learners. The first test, that is test A, was open and free whilst test B was diagrammatic in nature. There was no

statistical difference in their findings in the performance of learners in test A and test B. However, there was a dramatic difference on individual level.

Elia and Philippou (2004) also made an investigation on the functions of pictures in problem solving. The study was carried out in Cyprus. They used eight grade six learners. Their study was qualitative in nature. Their focus was on predetermined functions of pictures in Mathematics namely; decorative, representational, organisational and informational. They used a homogeneous group of high achievers in communication setting. Recently, Poch, van Garderen and Scheuermann (2015) carried out a study to investigate learners' understanding of diagrams for solving word problems in mathematics. They did their study in the United States of America and worked with learners with learning disabilities. Their work was both qualitative and quantitative. Thus, the use of diagrams in problem solving in mathematics has a long spanning history dating back to the age of early philosophers to the present day pedagogical approaches despite it receiving partial and divided consideration. This study was different from the previous studies in the sense that the main focus was to investigate the general roles of diagrams in problem solving in mathematics. The research design was also different from the previous studies in the sense that it adopted the survey and the quasi-experimental designs to gather and analyse data. On the other hand, quantitative and qualitative data sources were integrated to try and provide more convincing evidence to make generalisations. Furthermore, this study tended to differ with the previous studies undertaken in the sense that learners of mixed abilities were used in the study since Chifedza High School where most of the study was done, enrols learners from four units at grade seven to thirty six units. Thus, a more representative sample was used in this study in a bid to increase internal and external validity. Therefore, the research findings enabled the researcher to see beyond the horizon. This study took the task to investigate the role of diagrams in problem solving in mathematics as a tool to get understanding, to show understanding, and to apply what has been understood.

1.2 STATEMENT OF THE PROBLEM

The process of teaching and learning has been focused mainly on theoretical and abstract conceptualisation based on the rigorous drill embodied in rote and lecture methods of learning. These methods of learning though they allude to the use of media and teaching aids, they do not allow active involvement in the learning process. Low mathematics pass rate nationally is a cause for concern. Most students cannot model word problems into diagrams. This individual weakness affects the individual, the school, the district, the province and the nation at large, hence the need investigate and place the role of diagrams in problem solving in mathematics. In mathematics, most concepts are considered to be abstract, however, it is to the contrary due to the fact that all of them have an outstanding iconic and concrete representation which is well developed. Since every mathematical concept no matter abstract, can be graphically represented by numbers and in some cases by letters, for instance in algebra. Word problems appear to be challenging to most learners, therefore, teachers end up diagrammatically representing these problems in a bid to move from simple to complex. In trying to solve word problems, most learners fall tempted to make sketch diagram to represent a given problem, thus trying to make meaning out of a given situation. The problems encountered and attempts to devise plans have prompted the researcher to do this research. High failure rate in mathematics, which is a national concern, traditional teaching and learning strategies and iconic and concrete diagrammatic appeal were some of the key issues worth researching. Apryl et al (2015: 153) pointed out that, “A visual representation such as a diagram can be a powerful strategy for solving mathematical problems.” Thus, taking this into consideration, then, diagrams can be used to clear any discrepancies which words cannot sometimes clarify. It was a sincere intent of this investigation to establish the role of diagrams in problem solving in mathematics.

1.3 RESEARCH OBJECTIVES

The investigation was carried out with the following objectives in mind:

1.3.1. Establish the role of diagrams in helping learners to understand mathematical problems

1.3.2. Ascertain the role of diagrams in helping learners come up with plans and strategies that can be used in solving mathematical problems

1.3.3. Show the role of diagrams in helping learner apply and implement devised strategies use to solve problems in mathematics.

1.3.4. Outline the role of diagrams in helping learners to review, reflect and verify solutions obtained in solving problems in mathematics.

1.4 RESEARCH QUESTIONS

The questions this study was mainly focused on answering were:

1.4.1. What is the role of diagrams in helping learners to show understanding of mathematical problems?

1.4.2. What is the role of diagrams in helping learners to devise and design plans for solving mathematical problems?

1.4.3. What is the role of diagrams in helping learners in implementing devised plans and strategies in solving mathematical problems?

1.4.4. What is the role of diagrams in helping learners to review, reflect and verify solutions obtained in solving mathematical problems?

It was in the hope and focus of this investigation to answer these research questions so that the role of diagrams in solving mathematical problems can be established.

1.5 SIGNIFICANCE OF THE STUDY

The overall intention of this study was to improve the performance of learners in mathematics at ordinary level by placing learners and educators into a clear picture as to the role of diagrams in problem solving in mathematics. This further extended to cover on how, where and when diagrams should feature as 'cognitive' tools in various teaching and learning methodologies and techniques.

- **Learner**

The study was important to the learner as improved problem solving skills by widening the approach and scope of the basic stages involved in problem solving, that is, the use of graphical presentation in; understanding the problem, devising a plan to solve the problem, implementing the plan and in reviewing and verification of results. Improving problem solving capacity on its own was one of the major competencies in the updated curriculum in Zimbabwe. The research was also aimed at boosting the performance of learners in Mathematics through modified teaching and learning strategies.

- **Teachers**

The project was significant to teachers as it opened up the role for the inclusion of diagrams in problem solving. It also helped educators to shift from traditional methods of teaching and learning to modern methods which are child centred. The research also reduced the workload of teachers to that of a facilitator, for learning will be from concrete to abstract, simple to complex.

- **Ministry of Primary and Secondary Education**

The Ministry of Primary and Secondary Education was also a beneficiary in the sense that the study also held some practical benefit to the teaching and learning process as it provided an objective dimension to curriculum planning, implementation and innovation. This was so because it afforded the active involvement of the learners in refinement of problem solving techniques. This was as a result of measurable efforts from learners on which various tests and treatments were administered in line with the study. The researcher prepared research tools in form of tests and questionnaires. Comprehensive data analysis methods were applied, for example, various inferential statistical tests like hypothesis testing, χ^2 tests and correlation analysis, to validate the results and findings.

- **Textbook writers**

This study on the other hand helped textbook writers to include a lot of diagrams to enhance problem solving in mathematics. On the other hand, the writers improved on the quality of diagrams to include and on the relevance and purpose of the included diagrams.

- **Field of Research**

This study was worthy the researcher's time, effort and expense in that it provided the platform for the researcher to present well-researched and scholarly information in line with the topic. This did not only benefited the researcher but also the field of research by opening up, adding on and updating on what has already been contributed towards the role of diagrams in problem solving. The researcher was also afforded quality time with the learners, research supervisors and other concerned parties as they directly and indirectly got engaged in a scholarly discourse investigating problem solving and the role of diagrams in various steps of problem solving.

In a nutshell, the study held great significance in nearly all walks and important areas of life including all curriculum stakeholders and the scholarly field of research. It also assessed and

improved important teaching and learning methodologies and techniques. Finally, it also brought new contributors and fresh contributions in field of research.

1.6 DELIMITATIONS OF THE STUDY

The research was carried out in Chivi District at Chifedza High School since the school enrolled high, middle and low achievers, thus, mixed ability learners. The research was done in this remote rural area which was mostly shunned by most researchers. This research accorded the teachers and learners an opportunity to take part in research work. This research also excluded other areas due to financial constrains and the time factor. The study was delimitated to investigating the role of diagrams in problem solving in mathematics by form four learners as the researcher discovered that there was a gap in problem solving in mathematics and wanted to place the role of diagrams. The study was also delimitated to form four formal and non formal mathematics students since this was the senior group of students at secondary school which had experienced word problems and diagrammatic problems hence could place the role of diagrams in problem solving in mathematics. The research was carried out from January to October before the learners sat for their final examinations.

1.7 LIMITATIONS OF THE STUDY

The researcher faced financial challenges to fund the project since the researcher was self sponsored and had other financial commitments as the breadwinner. Funds permitting, the research could have been spread to different schools and districts in Masvingo Province. To curb this problem, the researcher started an income generating project to help him fund the carrying out of the project at the selected school. The researcher was situated at a school in a remote area

where there were no library facilities and therefore faced challenges of resources such as textbooks. However, he did away with the challenge by constantly travelling to Zvishavane to access the Midlands State University library from which a wide range of textbooks on the subject were available. On the other hand, the researcher used the internet to get some of the latest information on the subject. The researcher achieved this through the use of smart phones with PDF readers. Time was limited since the researcher was a classroom practitioner, so the researcher used some of the learners from his classes as a sample.

1.8 DEFINITION OF KEY TERMS

This provided the definitions of key terms used in this investigation in context of the study. That means the key terms were defined in narrow or broad sense as they were used in this particular study. Key terms defined in this study were:

DIAGRAMS-Smith (2003) defines a diagram as something that stands in place of something else, such as the expression of a concept. This is to say diagrams are pictorial representations of real objects and of an idea or concept showing the important features of the represented aspect. According to Diezmann and English (2001), diagrams are visual representations that display information in a spatial layout. On the other hand Veriki (2002) says that diagrams are considered structural representations in which the surface details are not important and this is their main characteristic and differentiation from pictures and drawings. A diagram in this study was a graphical outline that showed parts of something, how it worked and a drawing explaining the course of a process, that is to say, a diagram served to represent the structure of a problem.

A PROBLEM- Hornby (2010:486) defines problem as, “something difficult to deal with or understand” and “something that has to be done or answered.” A problem therefore is a challenging situation difficult to deal with or understand but, for which a solution has to be found.

In this study, a problem was any challenging situation that seemed not to have an immediate solution.

PROBLEM SOLVING- Problem solving according to Polya (1957) is a process with systematic phases which help the solver to arrive at a solution involving identification, understanding, and devising strategies, applying the strategies and verification and reviewing the solution to a certain problem. In other words, problem solving is a novel way to arrive at a solution to a problem. In this research, problem solving was a process by which individuals followed systematic steps in trying to find a solution to a given problem.

1.9 CHAPTER SUMMARY

This chapter introduced the aim of the study. The assumptions for the role of diagrams in problem solving in mathematics were reviewed, objectives were clearly stated, and sub-problems were stated in the form of questions. The significance of the study were clarified, delimitations and the limitations of the study were identified and justified. Key terms in the research study were defined. The next chapter focused on review of related literature.

CHAPTER TWO: REVIEW OF RELATED LITERATURE

2.0 INTRODUCTION

This chapter explored most of the work so far done on problem solving, problem solving in mathematics and the role of diagrams in problem solving in mathematics. This took the review from the classical philosophical and psychological backgrounds, through to contemporary background and recent scholarly work on problem solving and the role of diagrams in problem solving in mathematics. The first part explored the theoretical background on the literature on problem solving and the final part provided the conceptual and empirical literature on the role of diagrams in problem solving in mathematics.

2.1 THEORETICAL LITERATURE REVIEW

Learning theories are conceptual framework describing how information is absorbed, processed and related during learning. Cognitive, emotional and environmental influences, as well as prior experiences all play a part in how understanding of a world's view is acquired or changed and knowledge and skills retained. This research study was based on philosophical and cognitive theories.

2.1.1 PHILOSOPHICAL BACKGROUND

Classical philosophers like Plato believed and proposed that real knowledge unfolds from a series of pictorial representations that start as illusions then idols then real actors and then true knowledge and real knowledge of enlightenment of real world view. In his famous and elaborate Plato's Allegory of the Cave holds on the perspective that true knowledge begins as shadowy representation then images and finally real enlightenment. He further stressed that realisation of true knowledge is painful and sometimes may cause learners to withdraw from true knowledge. The central role of diagrammatic representations from the philosophical point has been emphasized towards acquisition of true knowledge and reality. Plato also included the knowledge of mathematics as truth and enlightenment. Thus, the importance of diagrammatic representation in acquisition of true mathematical knowledge can be traced back to the classical era. However, classical theorists based most of their inquiries on a wide range of disciplines and their studies lacked in-depth research strategies of the modern day. Their contributions towards problem solving in mathematics however, cannot be taken lightly. In this study, true knowledge and real knowledge were taken as a journey from pictorial representations that start as illusions, then idols, then real actors, that is to say, the role of diagrams in problem solving. Thus, this study explored the importance of diagrammatic representation in the acquisition of true and real knowledge making some reference to classical theories. Classical theories informed this study by understanding the problem, then devising plans and strategies which were diagrammatic representations and using the representations to get real knowledge. Thus, true knowledge should be obtained from reality.

Modern Humanist and Constructivist philosophers like Dewey (1933) advocate for learner centred and naturalistic activity based approaches to learning. Much work has been done up to this modern day coming up with proposals and suggestions for active involvement of learners towards problem solving. Tripathi (2013) states that, "researchers were influenced by the classical

work of Polya (1981) and Dewey (1933).” Polya did extensive work on problem solving in mathematics as a pedagogical process worthy to be acknowledged and applied. Polya proposed four stages in the process of problem solving and this study considered it worthy tribute to base its research questions and objectives on these four cyclic steps in problem solving in mathematics. Dewey’s 1933 philosophical theory of Pragmatism contributed invaluably towards learner centred and activity based problem solving approach to learning mathematics. The theory played a major role into the use of various learning aids in the teaching and learning of mathematics. However, just like their classical counterparts, their studies were mostly based on a wide range of disciplines as they focused on pedagogical strategies and techniques rather than on mathematics only as was the case with this study. Dewey’s theory was used in this study when learners were made to understand problems on their own, then devising plans and strategies and in implementing the devised strategies. Learners were actively involved throughout the learning episodes, thus, learning was child centred.

2.1.2 PSYCHOLOGICAL BACKGROUND

Classical behaviourist psychologists who include Ian Pavlov and B. F. Skinner used models and live animals to propose how behaviour can be stimulated and conditioned to effect learning. Their studies emphasized on visual stimuli and sensory conditioning to bring about behavioural changes, thus, learning taking place. Behaviourists advocated for the use of positive stimuli, reinforcements and environmental modification to bring about learning. From the behaviourist perspectives, it was noted that stimulating diagrammatic representation plays an important role in giving vivid simulations and stimulation of mathematical problem solving. However, studies done by Behavioural psychologists lack depth in the sense that most of the work was carried out on animals like cows, dogs and rats. Their findings therefore, cannot be taken to fully represent how human beings really learn. However, much can be tapped from the Behaviourists as learning is about change in behaviour and attitudes. Using the language borrowed from the Behaviourists,

this study sought to investigate the role of diagrams in stimulating and conditioning learners to understand, devise a plan, implementing the devised plan and reviewing their solutions in problem solving in mathematics.

Cognitive and learning psychologists who include Jean Piaget and Jerome Bruner proposed that human cognition and learning processes develop in stages or levels. Bruner (1960) says that learning proceeds through three levels namely; enactive, iconic, and symbolic phases. The iconic level which is a transitional link between the lower enactive level and the upper symbolic level according to Bruner (1960) is dominated by diagrammatic and graphical representations that enhance the process of learning. Piaget (1980) proposes that cognitive development proceeds in four stages namely; the sensory-motor, the preoperational stage, the concrete operational stage and the formal operational stage. The concrete operational stage in which concepts have greater meaning derived from concrete diagrammatic representations is the stage in which most of the learners at school level fall in. From Bruner's and Piaget's theories the emphasis is that learning of concepts should proceed from concrete to abstract and from simple diagrammatic representations to complex conceptualisation. However, Bruner and Piaget's theories are criticized for lack of in-depth research in the sense that Piaget used his own children in his studies and Bruner based his findings on cross breeding of social learning theorists' ideas and cognitive theorists' ideas. This research neither wished to draw its research subject from family members nor did it intend to crossbreed ideas from other related studies. However, its intention was to investigate the role of iconic and concrete graphical and diagrammatic representations in problem solving in mathematics. Mathematics textbooks, for example, Channon et al (2012)'s New General Mathematics 3 was dotted with diagrams on almost all of its 301 pages. Lewis's (2011) Focus on Mathematics Students' Book Four had almost 95% of its 396 pages dotted with diagrams and graphical representations. Although this evidence did not justify and explain the role of diagrams in problem solving in mathematics, it showed that diagrams were a prominent

feature and tool in the teaching and learning of mathematics. The ZIMSEC green book text of past Examination Mathematics Question Papers for 2008 to 2010 made use diagrams to pose questions and also to clarify questions in almost all of its examination items. This study therefore, took the task of investigating the role of diagrams in problem solving in mathematics.

2.1.3. . GEORGE POLYA'S FOUR STEP MODEL OF PROBLEM SOLVING

The investigation was closely guided by George Polya's four step model of Problem-solving modified and restated to give the problem statement. Polya (1957) proposed the following steps in problem solving;

1. Understand the problem

2. Devise a plan

3. Carry out the plan

4. Look back.

Sfard (1991) describes this process of developing meaning as consisting of 'internalisation' through acting new ideas with some process that makes them familiar and meaningful. This further translates to understanding and expressing these processes diagrammatically and graphically. Therefore, this investigation did not move away from these steps but rather sought to establish the following:

i. The role of diagrams in understanding mathematical problems

ii. The role of diagrams in devising a plan to solve mathematical problems

iii. The role of diagrams in implementing the devised plan in solving mathematical problems

iv. The role of diagrams in reviewing and reflecting on the solution in solving mathematical problems.

2.2. CONCEPTUAL MEANING OF PROBLEM SOLVING

Schoenfeld (1992) alludes the fact that problem solving in mathematics is a process where learners meet problems which they do not have an immediate means or formula that they can directly use to arrive at or move towards a solution. Tripathi (2013) explores problem solving in mathematics as a tool for cognitive development. In his work, Tripathi (2013) used some tests to find out how problem solving in mathematics enhance mental capacities and attitudes of learners towards mathematical reasoning. He was much concerned with the time taken by learners to achieve tangible results through problem solving. His findings were that problem solving in mathematics improves the proficiency in mathematical reasoning in learners who are above average in their academic performance but may cause fear and anxiety in the average and slow learners if over used. In this study the concept of problem solving was important in the sense that it formed the backbone of the study. Understanding problem solving rested on the mental prowess of understanding the problem which unlocked the doors to devising plans and strategies and implementing the devised plans, thereby solving the problem.

2.3 THE ROLE OF DIAGRAMS IN HELPING LEARNERS TO SHOW UNDERSTANDING OF MATHEMATICAL PROBLEMS.

Pantziara, Gagatsis and Pitta-Pantazi (2004) carried out a similar study in Cyprus in 2004. Their study was focussed on investigating the role of diagrams in solving non-routine problems in mathematics. The focus of this study was different from that of Pantiziara, Gagatsis and Pitta-

Pantazi in the sense that this study sought to investigate the role of diagrams in solving problems in mathematics, routine or non-routine. Pantziara et al (2004) used a quantitative research design. Their work involved the use of two tests, test A and test B. They administered the tests on 194, twelve year old learners. Test A was open and free while test B was diagrammatic in nature. Their work could have been also qualitative to add value to their findings. This study was both quantitative and qualitative in nature and was diagnostic in nature to increase credibility. In their findings there was no statistical difference in the performance of pupils in test A and test B. However, on individual level there was a dramatic difference, most learners who did well in test A did not do well in test B. Pantziara, et al (2004)'s study had a loophole in that, it was evident they used a non-homogeneous group in terms of academic abilities. However, the in depth study cannot be totally disregarded as it revealed that the role of diagrams in solving word problems in mathematics differs amongst learners of different abilities. The knowledge above assisted this research in designing a research study which was credible by borrowing the methodology, that is, the use of two tests and made some improvements on the weaknesses.

2.4. THE ROLE OF DIAGRAMS IN HELPING LEARNERS TO DEVISE AND DESIGN PLANS FOR SOLVING MATHEMATICAL PROBLEMS.

Elia and Philippou (2004) also made an investigation in Cyprus in 2004 on the Function of Pictures in Problem Solving. Their investigation was focussed on the function of pictures in problem solving whilst this study focussed on investigating the role of diagrams in problem solving in mathematics. Their study used eight Grade 6 learners as their subjects in case. Their study was qualitative in nature as they focused on four predetermined functions of pictures in mathematics namely; decorative, representational, organisational and informational. The study worked with a homogeneous group of high achievers in communication setting. They developed tasks and interviews for gathering qualitative data. The qualitative nature of the study compromised the findings and conclusions of the research. A research study should be both

quantitative and qualitative in nature to increase its credibility. The subjects used also compromised the findings and conclusions of the study; eight is too small a number. In their study, they discovered that pictures only serve representational, informational and organisational functions in solving mathematical problems. The decorative function was of no use to the process of problem solving in mathematics. They further suggested that the use of pictures in successful mathematical problem solving depend on the relationship between the picture, the task and the learner's mental abilities. Carney and Levin (2002) also supported their view that pictures do not serve a decorative function in problem solving in mathematics. Elia and Philippou's (2004) study however, used a small group of 8 grade 6 pupils only and their work was a qualitative study which was sometimes more or less subjective. This study increased the number of subjects and used both a quantitative and qualitative approach to increase on credibility.

2.5.THE ROLE OF DIAGRAMS IN HELPING LEARNERS IN IMPLEMENTING DEVISED PLANS AND STRATEGIES IN SOLVING MATHEMATICAL PROBLEMS

Pantziara, Gagatsis and Pitta-Pantazi (2004) carried out a similar study in Cyprus in 2004. Their study was focussed on investigating the role of diagrams in solving non-routine problems in mathematics. The focus of this study was different from that of Pantiziara, Gagatsis and Pitta-Pantazi in the sense that this study sought to investigate the role of diagrams in solving problems in mathematics, routine or non-routine. Pantziara et al (2004) used a quantitative research design. Their work involved the use of two tests, test A and test B. They administered the tests on 194, twelve year old learners. Test A was open and free while test B was diagrammatic in nature. Their work could have been also qualitative to add value to their findings. This study was quantitative and qualitative in nature and also diagnostic in nature to increase credibility. In their findings there was no statistical difference in the performance of pupils in test A and test B. However, on individual level there was a dramatic difference, most learners who did well in test A did not do well in test B. Pantziara, et al (2004) had a loophole in their study in that, it was evident they used

a non-homogeneous group in terms of academic abilities. However, the in depth study cannot be totally disregarded as it revealed that the role of diagrams in solving word problems in mathematics differs amongst learners of different abilities. This research borrowed the use of tests which was a quantitative data source. The knowledge above assisted this research in designing a research study which was credible by borrowing the methodology and making some improvements on the qualitative aspect through the use of questionnaires.

2.6. THE ROLE OF DIAGRAMS IN HELPING LEARNERS TO REVIEW, REFLECT AND VERIFY SOLUTIONS OBTAINED IN SOLVING MATHEMATICAL PROBLEMS

Poch, van Garderen and Scheuermann (2015) carried out a research study in the United States of America in 2015. They worked with pupils with learning disabilities to investigate learners' understanding of diagrams for solving word problems in mathematics. The focus of their work was towards an establishment of a supporting framework for promoting the use of diagrams for solving word problems in mathematics by special educators. They worked on five aspects namely; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Their study had a two level setting which developed a checklist for assessing diagram proficiency and another checklist for checking conceptual understanding. This study however, sought to investigate the role of diagrams in problem solving in mathematics. Their work was both qualitative and quantitative and seemed to be in-depth except for that it was based on pupils with learning disabilities and problem solving on word problems only. This research study borrowed the use of both quantitative and qualitative approaches and was diagnostic in nature. However, their work was prescriptive rather than diagnostic, because it attempted to give diagrams a place in the process of problem solving in mathematics. In their study, they found out that diagrams play a pivotal role in solving word problems by giving them

meaning. Thus, diagrams helped learners to review, reflect and verify solutions obtained in solving mathematical word problems. The above knowledge assisted this research study by adopting the quantitative and qualitative nature and also adding the diagnostic aspect.

2.7. CHAPTER SUMMARY

The use diagrams as concrete cognitive development tools in problem solving in mathematics has a long spanning history dating back to the age of early philosophers to the present day pedagogical approaches. However, research work on the role of diagrams in problem solving in mathematics has received partial and divided consideration. This may be due to the fact that the over assumption that diagrams are an indispensable media tool in mathematics which is widely used in mathematics textbooks and question papers. This study took the task to investigate the role of diagrams in problem solving in mathematics as a tool to get understanding, to show understanding, and to apply what has been understood. The next chapter focused on research design and methodology.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

3.0 INTRODUCTION

This chapter provided an overview of the strategy to be used to conduct the research and collect necessary information. It entailed choice of research approaches and techniques applied to corresponding research objectives. It also covered the research design of the chosen framework. The sampling techniques, procedures, as well as an outline of the data collection methods that were employed were explored including the pros and cons of using each of the preferred methods. Thus, the focus of this chapter was to specify and describe in detail the research methodology the researcher used to evaluate the role of diagrams in problem solving in mathematics at ordinary level at Chifedza High School.

3.1 RESEARCH DESIGN

According to Nieswidomy (1993), a research design is a plan of how a particular study is going to be conducted, mainly concerned with the type of data to be collected and the means used to obtain

the data. Oswala (2001) says that research design refers to the overall plan to use and follow in answering the research questions. In other words, a research design is a strategy to be employed in a bid to effectively address the research problem. Therefore, the function of a research design in this study was to ensure that evidence obtained enabled the researcher to effectively address the research problem in a logical and unbiased manner in a bid to increase internal and external validity.

On the other hand, Kumar, Aaker and Day (1999) define research design as a detailed blueprint to guide a research study towards its objectives; thus, the process of designing a research study involves many interrelated decisions on data sources, research approaches, research instruments, sampling plan, and contact methods. They identify the most significant decision as the choice of the research approach because it determines how the information will be obtained.

This study adopted the survey (by questionnaires) and quasi-experimental (by tests) designs in which mixed methods (quantitative and qualitative) were used to gather and analyze data. The quantitative techniques were conveniently chosen to dominate over the qualitative ones. The relationships between the variables affecting the role of diagrams were analyzed quantitatively by means of statistical tests (Z-test). Sometimes qualitative data is difficult to justify with statistical tests, and so the quantitative data gathered from tests were used to verify and corroborate data gathered from opinions, beliefs and suggestions of the respondents (through questionnaires). Data gathered from the review of related literature (secondary data) was used to cater for the issue of triangulation in order to overcome threats to validity and reliability of instruments. It was hoped that such a design would produce authentic, valid, general and reliable information relating to the role of diagrams in problem solving in mathematics. Another purpose of choosing this design was to integrate the two (quantitative and qualitative) data sources and try to provide more convincing evidence relating to the role of diagrams in problem solving in mathematics.

3.2 POPULATION

Crouch and Housden (1996) say that population is used in the research sense, meaning the whole group under consideration. Wallen (1996) says that it is upon the population that the results of the study are generalized. According to McGrown (2001), population is the totality of persons, events, organizational units, case records or other sampling units from which the sample is selected and with which the research problem is concerned. On the other hand, Kahn (1993) defines population as any group of individuals or an item that has one or more characteristics in common that are of interest to the researcher. The research population of this study was all the teachers and form four learners at Chifedza high school, formal or non-formal. The population was conveniently selected. According to Dhliwayo and Keogh (2002), convenient sampling is a sampling technique in which the researcher selects items that are readily available. Form four learners were my target population because they were the most senior learners in secondary school and learners at Chifedza High School were of mixed abilities. On the other hand, mathematics and science teachers at this school were of different qualifications and experiences. It was the researcher's view that the sample chosen was representative of the population to enable the researcher to see over and beyond the horizon.

3.3 SAMPLE AND SAMPLING PROCEDURES

A sample is part of a population. The sample selected should reflect the characteristics of the population in order to derive information reflecting the whole population. It is often impractical for the researcher to study the entire population, hence the need for samples. Jancowicz (1995) defines sampling as a process of selecting a few (a sample) from a bigger group (the population) to become the basis for estimating or predicting a fact, situation or outcome regarding the bigger group. Crouch and Housden (1996) further describe a sample as a limited number from a large

group for testing and analysis, on the assumption that the sample can be taken as a representative of the whole group.

According to Crouch and Housden (1996), good sampling methods allow a high degree of precision in estimating the results which would have been achieved from a census. Supporting this view, Zikmund (1997) points out that if properly selected samples are sufficiently accurate in most cases. Sampling techniques can be narrowed down to two broad types namely, probability and non-probability sampling. Shajaham (2005) says that according to the law of chance, probability sampling provides a scientific technique for drawing of samples from the population where each unit in the universe has some definite pre-assigned probability of being selected into the sample. Tull and Hawkins (2000) define a probability sample as one in which sampling units are selected by chance and for which there is a known chance of each unit being selected. On the other hand, non-probability (non-random) sampling is such that the probability of each case being selected from the total population is unknown and cannot answer questions that require inferences about the population characteristics.

Four sampling techniques were used in this study. These were convenience, simple random, stratified and purposive sampling.

Chifedza High School

The researcher conveniently chose to carry out this study at Chifedza High School in Chivi District where he worked as a secondary school teacher. Since the aspect of researcher bias cannot be overruled in convenience sampling, this limitation was minimized using a variety of data collection methods and instruments (triangulation).

Formal learners

According to Jancowicz (1995), in most populations in research, there exists very little similarity among elements and the situation dictates that a sub-set (sample) of the population be used to estimate the overall responses of a population. Wegner (1993) says that the sample findings will be used to deduce the likely overall behavior of a random variable under study. The probability sampling technique was employed in the study. Wegner (1993) says that probability sampling includes all selection methods where the observations to be included in the sample have been selected on a purely random (chance) basis from the population. It is only through the random selection of sampling units from the population to be included in a sample that sampling errors can be measured and, the researcher is able to establish the representative nature of the sample drawn. The simple random sampling technique was used in this study. Shajaham (2005) defines simple random sampling as a technique in which a sample is so drawn that each and every unit in the population has an equal and independent chance of being included in the sample. Krueger (1994) echoes this when he describes simple random sampling as a method where the units are selected from the population in such a manner as to afford every unit of the population the same chance of being selected. The advantage of simple random sampling was that of affording each unit a chance to be selected, thus reducing the amount of error or probability that the sample units would not be a true representation of the population. It was a disadvantage in that it was based on an assumption of uniformity with the population being represented. Stratified random sampling was used to select O' level Mathematics students. In stratified sampling, population is first divided into groups or layers called strata. The items to be selected are such that they are as heterogeneous as possible between strata and as homogeneous as possible within strata. The three O' level mathematics classes divided the learners into groups. Students were naturally grouped in terms of high, medium and low achievers.

Teachers

A non-probability sampling procedure was used for teachers and non-formal learners. Wegner (1993) defines non-probability sampling as any method in which the observations are not selected randomly. Selvam (2004) says in non-probability sampling, there may be instances that certain units of the population will have a zero probability of selection because judgments, biases and purpose of interviewers are considered to be the criteria for selection of sample size of such sampling. One such example was purposive sampling which was the sampling technique used in selecting teachers and non-formal learners. According to Shao (1999), purposive sampling involves selecting sample items hoped to provide the needed, authentic and reliable information. Selvam (2004) propounds that purposive sampling is a method in which the researcher will decide on the choice of sampling units based on their purpose. Purposive sampling was used to select teachers where mathematics teachers were selected to get as much needed, authentic and qualitative data as possible (purposive).

Non-formal learners

Automatic sampling was used on non formal learners since there were only five non formal learners at Chifedza High School, therefore their choice was automatic.

The sampling frame for this study comprised of formal and non-formal form four students and teachers at Chifedza High School in Chivi District. The sample was made up of twenty five (25) formal form four students, five (5) non-formal form four students and ten (10) teachers.

3.4 RESEARCH INSTRUMENTS

Research Instruments are tools used for collecting information and data needed to find solutions to the problems under investigation. Examples of tools include tests, questionnaires, interviews and observation guide.

3.4.1 Questionnaires

Crouch and Housden (2000) define a questionnaire as a research instrument comprising of a combination of questions that are filled by a sample population from which information is sought. The questionnaires were used to gather information from 10 teachers, twenty-five (25) formal students and five (5) non-formal form 4 students. Pre-set questions were used to acquire information through respondents written responses. In order to gather various views from the respondents, a series of open and closed ended questions were used. The researcher used questionnaires since they are cheap and also they allow sample units ample time to respond to questions, thus, fully expressing themselves. Open ended questions were used to allow respondents an opportunity to express their opinions, ideas and line of thinking regarding the questions in their own words since there is no predetermined list of responses to aid or limit the respondents. On the other hand, closed questions were used since they reduce the amount of thinking and effort required by the respondents and also reducing interviewer bias because the respondents marked their answers in a box.

However, according Oppenheim (1992), open ended questions can generate large amounts of data that can take a long time to process and analyze, which is a limitation on the use of questionnaires. To curb this, the researcher designed the questionnaires with a limited space so that responses were concise. Furthermore, another limitation put forward by Gilbert (1993) is that, respondents may answer superficially especially if the questionnaire takes a long time to complete. However, this research study avoided the common mistake of asking too many questions.

Wilson and McClean (1994) say that generally it is relatively quick to collect information using a questionnaire and potentially information can be collected from a large portion of a group. On the other hand, questionnaires are more objective, certainly more so than interviews because the responses are gathered in a standardized way. In this research study, a large sample of respondents was reached and anonymity of respondents was assured, generally resulting in more honest responses. Respondents were more willing to answer personal questions in a free manner which was an advantage in this research study. Questionnaires were hand delivered, for closer follow up and further discussion on matters that require clarification between the researcher and respondents. In this study questionnaires for teachers and learners were used to collect data from the respondents. The questionnaire guaranteed confidentiality, which in turn generated more valid and reliable information from the respondents. The questionnaires contained both closed and open- ended items. The respondents were informed about the purpose of the study, the importance of their contributions and were assured of confidentiality in terms of the information to be provided.

3.4.2 Tests

Borg and Gall (1992:246) say, "A test is any structured set of questions, a situation that can be analyzed to yield numerical scores for which inferences can be made on how individuals differ in the construct measured by the test." This means that after a test has been written, the teacher sits down to analyze how learners would have performed. According to Tichapondwa (2000), a test is an instrument that is used by teachers to promote a goal to be reached by students and promote sense of achievement to confirm what has been learnt. He goes on to say that, a test reveals weaknesses in instruction. In this research, two tests were used by the researcher to compare the performances of the 25 formal and 5 non-formal form four level students when using diagrams and when not using diagrams in problem solving in mathematics. The study used two tests in which two types of questions namely descriptive questions in test 1 and diagrammatic questions

in test 2. Descriptive questions were used in test 1 with word problems which needed to be interpreted into diagrams and then solutions follow. Diagrammatic and graphical questions with diagrams were used to pose questions in test2. The purpose of administering the two tests was to compare the performance of learners in the two tests so that the researcher would be able to place the role of diagrams in problem solving in mathematics.

The use of tests in this study was advantageous in the sense that tests are often standardized (i.e., the same stimulus is provided to all participants), thus allowing comparability of common measures across research populations. Furthermore, tests could be administered to all groups at once which save time. However, Watson (1998) says it is expensive to produce the test for each participant and there are high chances of non response to some items on the test. This research study made use of tests that provided criterion referenced results. The researcher made an assurance that such tests were only one aspect of a multi-method approach in which no firm conclusions based on norm-referenced data could be reached without cross validation from other sources (triangulation), in this research, questionnaires.

Tests are usually already developed and normally measure what they are intended to measure, that is to say, there is validity and reliability in the use of tests that brings credibility on the part of the research work. Furthermore, a test is objective, relatively and easy but useful mostly where group level performance and external comparisons of results are required. Tests are easy to administer and mark and immediate feedback can be given, that is to say, it is easy to analyze and make conclusions at the earliest convenience.

3.5 .DATA COLLECTION PROCEDURES

Borg and Gall (1990) define data collection as a process that involves gathering of data or information through all data collection techniques. The researcher designed research instruments, questionnaires and tests and sought approval from the supervisor. The researcher obtained a

clearance letter from Midlands State University. The researcher presented two copies of each of the following to the Provincial Education Director (Masvingo Province), clearance letter from MSU, student ID, application letter, questionnaires for teachers and learners, test 1 and test 2. The PED issued a covering letter which was attached to the sets of documents. One set was taken by the researcher to Harare to the Permanent Secretary (Ministry of Primary and Secondary Education) who issued a clearance letter which allowed the researcher to carry out an educational study at Chifedza High School. The researcher passed through the PED's office again where the clearance letter was stamped, then taken to the District Schools Inspector (DSI) Chivi District where it was again stamped. The researcher took the letter to Chifedza High School where the research was conducted.

In addressing key issues about research instruments: validity, reliability and objectivity, the questionnaires and tests were pre-tested before administering them to the sample. Ten learners and five teachers were involved in the pilot study. This was done to bring out any vague or poorly phrased questions and also to indicate whether the instructions to the respondents were clear. This was done two weeks before the actual research study to allow the researcher ample time to make some amendments where necessary so that the work would be credible.

3.5.1 Sources of data

Data was a vital input for this research and was collected from primary sources and secondary sources. Collection of primary data involved collection of data from individuals who altogether constitute the population. Since at most times it is nearly impossible to collect it from the entire population, a part of the population (sample) is used to represent the population. On the contrary, secondary data used in this research contained information which already existed in some form or another but which was not initially collected for the purpose of the research at hand. In this study,

the researcher used 2 tests, 2 sets of questionnaires and text book diagrams. The researcher used questionnaires to collect data from twenty-five form four formal pupils, five non-formal students and ten teachers at Chifedza High School in Chivi District. Two tests were administered to twenty-five formal and five non-formal form four learners at Chifedza High School. Primary data was important in this research because data collected was free from bias since it was first-hand information. Primary data obtained was fresh gathered for a specific project or research which enabled the researcher to obtain information concerning the study area.

However, there were some limitations to collecting primary data. It was time consuming to gather information and at the same time expensive to collect data considering the resources channeled towards the research project. To counter the disadvantages of collecting primary data, the researcher minimized the costs by arranging a single day for the distribution of questionnaires, and a single day for their collection, so as to reduce frequent visits that would otherwise result in an increased time cost of data collection. Likewise, the tests were administered as single day tasks, i.e. test 1 was administered on a single day, so was test 2. On text diagrams and graph count inspection, diagrams and graphs used in the following tests were counted numerically; New General Mathematics Book 4 by Channon et al (2011), Focus on Mathematics book 4 by Lewis (2012) and the ZIMSEC Mathematics 4008/4028 Green book 2008 to 2010 O'level past examination papers. Participants took part in the research by their own choice. According to Chiromo (2006), participants should not be forced to take part in the study but should do so with consent. As a means of protecting the confidential treatment of data from students and teachers, all completed questionnaires were sent without names and the tests were written without writing down names of pupils.

3.6. VALIDITY, RELIABILITY AND OBJECTIVITY

Bollen (1989) defines reliability as consistency of measurement. On the other hand, Nunnally (1978) defines reliability as stability of measurement over a variety of conditions in which basically the same results should be obtained. Nunnally (1978) further says that reliability is that part of a measure that is free of purely random error and that nothing in the description of reliability requires that the measure be valid, therefore, it is possible to have a very reliable measure that is not valid. However, reliability is necessary but not a sufficient condition for validity.

AERA, APA, & NCME (1999:9) define validity as "the degree to which evidence and theory support the interpretations of test scores entailed by the proposed uses of a test". Validity is related to the "proposed uses" of the scores.

Validity, reliability and objectivity are the benchmarks on which research instruments are measured. Fraenkel and Wallen (1996) state that the validity of an instrument must always be considered within the context inferences the researcher makes regarding particular areas or topics. In other words, the researcher needs instruments that would allow one to validate conclusions about the characteristics (perceptions and attitudes) of the individuals under study.

In addressing key issues about research instruments: validity, reliability and objectivity, the questionnaires were pre-tested before administering them to the sample. This was done to bring out any vague or poorly phrased questions and also to indicate whether the instructions to the respondents were clear. Below are the discussions on the research instruments to be used by the researcher.

3.7. ETHICAL CONSIDERATIONS

The accepted principles of good research ethics according to Cohen and Manion (2012) encourage good relationships between the researcher and the participants. To handle the relevant

and legal implications, the researcher applied for the Ministry of Primary and Secondary Education permit to conduct the research at the selected school. The researcher also explained to the respondents the significance of the study and the contributions it would make to the overall education system, before administering the instruments. Furthermore, the researcher asked the respondents to respond willingly, truthfully and objectively and told them that they were free to withdraw at any given time. The researcher also assured the respondents that their right to confidentiality, anonymity, freedom of choice and expression would be adhered to. The researcher on the other hand got consent from parents or guardians for learners under 18 years old. The researcher considered that the purpose of this research was to collect data and not to influence the respondents' opinions on the Role of Diagrams in Problem Solving in Mathematics.

3.8 DATA PRESENTATION AND ANALYSIS

According to Zikmund (1997), once field work has been completed the data is converted into a format that answers the decision makers' questions. Data presentation and analysis is what the researcher does with collected data. According to Hitchcock and Hughes (1989), data analysis is what the researcher does with data collected so that theories and generalizations can be developed. The data was presented and analyzed according to how research questions were addressed. This research study was both quantitative and qualitative and also diagnostic in nature. Therefore, the researcher used tables, graphs and descriptions to clearly and vividly present the data for easy interpretation, analysis and inference. The information was presented using tables, graphs and written explanations to enable easy comparison and clear projection of the situation in line with the research questions. A statistical test was done to test for significance. A 90% confidence interval was done to test the null and the alternate hypothesis on the role of diagrams in problem solving in mathematics making use of collected data. The findings were laid out in Chapter four, together with their detailed discussion.

3.9. CHAPTER SUMMARY

The chapter gave an outline of the research design and the reason for choosing the design, population and sampling techniques used as well as how research instruments were developed. Research instruments, their strengths and weaknesses and the validity and reliability of such instruments also formed part of this chapter. Questionnaires and tests were used in acquiring accurate information regarding the Role of Diagrams in Problem Solving in Mathematics at Form four level. The next chapter focused on the analysis and presentation of results obtained and discussion of such findings.

CHAPTER FOUR: RESEARCH FINDINGS, DATA PRESENTATION AND DATA ANALYSIS

4.0 INTRODUCTION

This chapter presented data gathered from questionnaires, textbooks and the two tests administered. The first part presented biographic data on the research subjects, that is, data on learners' gender, form three mathematics results and text books that the learners used and had. The second part presented data on teachers who responded to questionnaires, that is, gender, academic qualifications and subjects of specialisation.The chapter focused on data presentation followed by analysis and discussion. The findings related to the research questionnaires and the tests given to both educational practitioners that are teachers and students. The main thrust of the study was to investigate The Role of Diagrams in Problem-Solving in Mathematics. Tables and other mathematical data presentation and analysis tools were used in this chapter.

4.1 DATA PRESENTATION AND ANALYSIS

Data was presented in two categories that is, personal details of respondents and responses of respondents to research questions.

4.1.1 Demographic data

To establish the general characteristics of the respondents involved in this study, question questions 1 to 4 were included in the questionnaire for learners as well as question 1 to 6 in the questionnaire for teachers. Tables 1a to 1g below give a summary of the characteristics of the respondents. The study sought to ascertain the gender of learners who participated. Table 1a provides a summary of the findings.

Table 1a: Distribution of formal and non-formal learners by gender (N=30)

	MALE	(%)	FEMALE	(%)	TOTAL	TOTAL

						(%)
FORMAL	11	37	14	47	25	84
NON-FORMAL	2	6	3	10	5	16
TOTAL	13	43	17	57	30	100

N=sample size

From the table it can be noted that the sample was made up of more female learners than male learners. This was due to the criteria of sampling the population from which the sample was drawn. It was biased towards females in that there are more female than male learners in form four at Chifedza High School in 2017. The sampling process was also slightly based on willing participants and on form three marks in mathematics. For non-formal learners all the five learners who participated make up all the learners doing form four as non-formal learners. The study however had no wish to base its findings on gender lines but felt it necessary to provide information on gender as it may be useful to other researchers and stakeholders who may find it useful in future and for further analysis.

An attempt was also made to find out the age distribution of the learners as contained in question 2 of the questionnaire for learners. Table 1b below gives a picture on the ages of the respondents.

Table 1b: Distribution of form 4 learners by age (N=30)

Age in years	Frequency	Percentage (%)
15-17 years	12	40
18-20 years	18	60
Above 20 years	0	0
Total	**30**	**100**

N=sample size

From the sample of 30 form four learners, the ages ranged from 15 to 20 years with a mean age of about 18, this reflect that the sample was made up of mature students at the formal operational stage, thereby students were able to analyze information provided and provide sound ideas. Due to limited time and resources, the researcher managed to carry out the research using a sample of ten teachers. To establish the general characteristics of the respondents, question 1 to 6 were included in the questionnaire for teachers. Table 1c to 1f give a summary of the characteristics of teachers involved in this study.

Table 1c: Distribution of Mathematics and Science teachers by gender (N=10)

	Mathematics	%	Science	%	Total	%
Male	4	40	3	30	7	70
Female	1	10	2	20	3	30
Total	**5**	**50**	**5**	**50**	**10**	**100**

N=Sample size

Ten teachers were involved in the data collection procedure in this study. The teachers were selected from the Mathematics department and Science department, with five teachers from each department. Teachers responded to questionnaires which were designed for collecting data for the study. The teachers contributed immensely towards the study. The teachers to which questionnaires were given teach mathematics and some had taught mathematics at

some point. The ages of the respondents were also established. Question 3 on the questionnaire for teachers was included to determine the ages of the respondents. Table 1d gives a summary of age distribution.

Table 1d: Distribution of Mathematics teachers by age (N=10)

Age in years	**Count**	**Percentage (%)**
25 years and below	0	0
26-30 years	2	20
31-35 years	5	50
36-40 years	2	20
41 years and above	1	10
Total	**10**	**100**

N=Sample size

From the table, it can be noted that most (80%) of the respondents were over 30 years which implied maturity and sound judgement, therefore their contribution in completing the questionnaire can be heavily relied upon. An attempt was also made to ascertain the academic and professional qualifications of the respondents.

Table 1e: Distribution of Mathematics and Science teachers by academic and professional qualifications (N=10)

Academic and/or Professional qualification	Count	Percentage (%)
Diploma in Education only	3	30
Diploma in Education + BED	6	60
Diploma in Education + BED + MED	1	10
Total	**10**	**100**

From the table above, it can be observed that the respondents in this study were professionals in Mathematics, hence understood the terms and language used in the questionnaire, that's increasing the validity of responses to the research questions. The study also sought to establish the teaching experiences of these respondents by including question 4 in the questionnaire. Table 1e below provides a summary on the findings.

Table 1f: Distribution of Mathematics teachers by teaching experience (N=10)

Teaching experience in years	Frequency	Percentage
0-5 years	1	10
6-10 years	2	20
11-15 years	4	40
16-20 years	2	20
21-25 years	1	10
26 and above	0	0
Total	**10**	**100**

Most (70%) of the mathematics teachers who took part in this study had more than 10 years teaching experiences. Only (10%) fall within 0-5 years. This implied that most respondents (70%) were well experienced in teaching the subject at ordinary level, hence were capable of

providing truthful information with regards to the role of diagrams in problem solving in mathematics.

The study also sought to establish the grades attained by the learners at form three level. Question 4 on the questionnaire for learners was included to find out the mark and grade achieved by the learner.

Table 1g: Distribution of formal learners by grade attained in mathematics at form three level (N=30)

Mark	**Grade**	**Girls**	**%**	**Boys**	**%**	**Total**	**%**
70-100	A	1	4	2	8	3	12
60-69	B	3	12	3	12	6	24
50-59	C	2	8	3	12	5	20
40-49	D	3	12	2	8	5	20
30-39	E	2	8	0	0	2	8
0-29	U	3	12	1	4	4	16
Total		**14**	**56**	**11**	**44**	**25**	**100**

N=Sample size

From the table of form three results it can be noted that the sample selected had sixteen learners having scored marks below 60%. This was done with a deliberate intend not to use subjects who are dominantly mathematically high performers nor low performers. High performers and low performers were however included in the sample of formal learners to account for some variables which may not be explained within the study and to ensure fair distribution and representation of the target population.

Table 1g was used in the data analysis as an initial point of comparison with results obtained from the two tests. The form three results were used as a control to check if there was any significant change in performance of learners after the administration of test treatments. This enabled the writer to check if diagrams play any significant role in problem solving in mathematics by comparing changes in learner performance. This distribution enabled the researcher to place the role of diagrams in problem solving in mathematics. The study also sought to find out the distribution of mathematics textbooks among learners. The questionnaire for learners included question number 10 which sought to establish the distribution of mathematics textbooks. Table 1h gives a summary of the findings.

Table 1h: Distribution textbooks to learners (N=30)

Textbook	**Frequency**	**Percentage (%)**
New General in Mathematics Book 4	18	60
Focus on Mathematics Book 4	6	20
Mathematics Today Book 4	3	10
Others	0	0
None	3	10
Total	**30**	**100**

Information on textbooks provided some insight on how learners are exposed to live diagrammatical representation of mathematical problems. Although data on textbooks seemed to be of limited use, it proved to be invaluable as number of diagrammatical representations have been enumerated in each text as a research procedure. This also helped to establish the role of diagrams in problem solving in mathematics. The next section documents the responses given by the respondents that try to address the research questions.

4.2. RESPONSES TO RESEARCH QUESTIONS

Responses to questions in the research instruments were arranged around the research questions as follows:

4.2.1 Responses on the role of diagrams in helping learners to show understanding of mathematical problems.

To address this research question, question 1 to 3 in section B were included on the questionnaire for teachers.

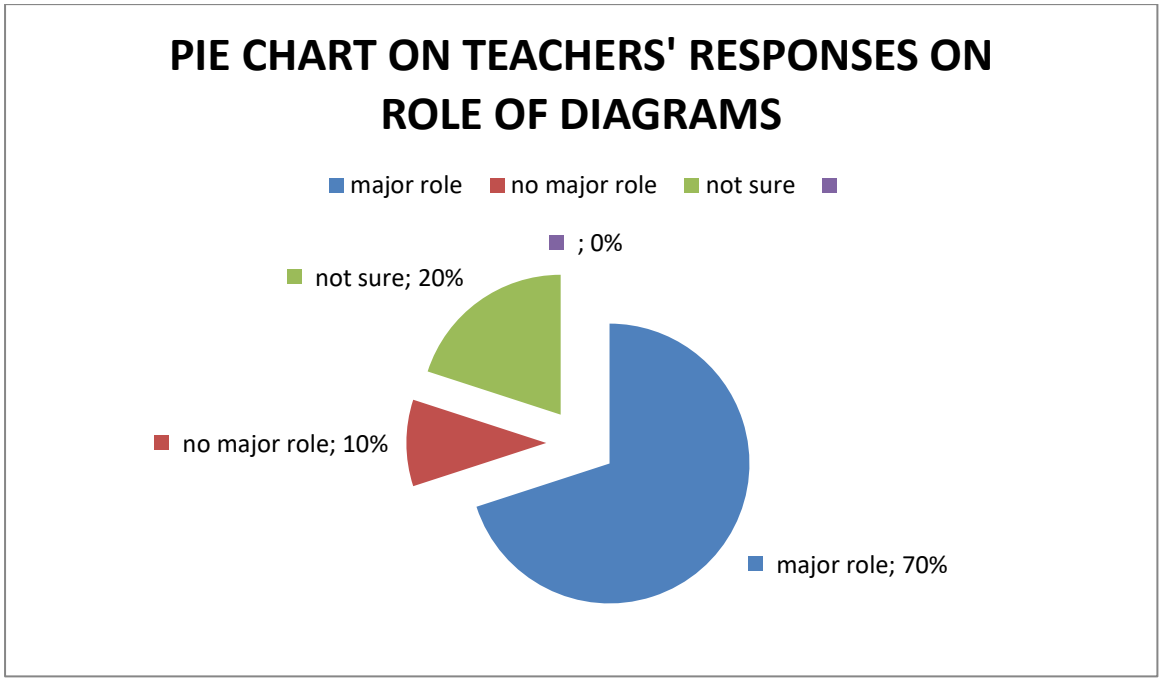

Figure 1: Distribution of teachers according to their responses on the role of diagrams

Seven out of ten teachers said that diagrams play a vital role in problem solving in mathematics at any level in mathematics. The respondents clarified their convictions and below are some of their contributions:

- *Diagrams fill in the gap which words may fail to clearly state*
- *Diagrams provide the means by which learners can communicate their solutions.*

- *Diagrams provide a pictorial reference which gives a holistic view of the problem.*
- *Diagrams help learners to overcome linguistic challenges which learners may face in understanding the problem.*

In general more than 70% of the teachers believe that diagrams play a vital role in problem solving in mathematics, with 10% of the teachers saying that diagrams do not play a major role in problem solving in mathematics. Their main concern was that diagrams sometimes complicate the problem as some teachers and learners draw poor and diagrams which are not proportionate.

The researcher went on further to ascertain the role of diagrams in helping learners devise and design plans for solving mathematical problems through research question 2.

4.2.2 Responses on the role of diagrams in helping learners to devise and design plans for solving mathematical problems.

To address this research question, question 1 to 4 in section C in the questionnaire for teachers was designed. Table 2a gives a summary of the responses.

Table 2a: Distribution of teachers by Teaching Methods and Techniques used

Teaching Method	Never	%	Sometimes	%	Often	%	Very Often	%	Always	%
Lecture and Rote	0	0	0	0	4	40	3	30	3	30
Problem Solving	0	0	2	20	0	0	1	10	0	0
Group Work	0	0	4	40	1	10	2	20	1	10
Learner Centred	0	0`	4	40	0	0	4	40	2	20

The teaching method which was used most often by teachers in their lessons is the lecture method. However, all the teachers were quick to acknowledge that this was not the best method but it has more advantages than others. It was pointed out that the lecture method was the least time and less resource consuming than the other methods which are more participatory and learner centred. The researcher went on to show the role of diagrams in solving mathematical problems through research question 3.

4.2.3 Responses on the role of diagrams in helping learners implementing devised plans and strategies in solving mathematical problems.

To get answers to the research question, questions 1, 2 and 3 in section D of the questionnaire for teachers and questions 4, 6 and 8 in the questionnaire for learners were designed. Table 2b shows the responses.

Table 2b: Distribution of lesson achievement using diagrams (N=10)

	Frequency	**Percentage (%)**
Excellent	8	80
Fair	2	20
Poor	0	0
Total	**10**	**100**

Eight of the teachers rated their achievements in lessons in which they used diagrams to be excellent with none giving a poor rating. Two teachers rated their lesson success with diagrams fair as they said that diagrams are difficult for them to draw and therefore they are time consuming especially if they are to be drawn on the chalkboard.

The researcher went further to further to outline the role of diagrams in helping learners review, reflect and verify solutions obtained in solving mathematical problems through research question 4.

4.2.4 Responses on the role of diagrams in helping learners review, reflect and verify solutions obtained in solving mathematical problems.

In addressing the research question, test 1 and test 2 were designed and administered. Test 1 had word problems with instructions that could be diagrammatically interpreted in the process of solving the problems. Test 2 was purely diagrammatic with each test item having a diagram or diagrams from which problems were based. Each test item was marked out of five with the following scoring guide;

Table 2c: The Scoring Guide on the Marking criteria on Test 1 and Test 2

Understanding and use of diagram	Devising diagrammatic plan	Diagrammatic manipulation to arrive at solutions	Solution related to diagram	Solution found through any means	Overall score	%
A	**B**	**C**	**D**	**E**		**F**

Test 1 had sixteen test items each with five marks therefore it was marked out of eighty. Test 2 had twenty test items each with five marks therefore it was marked out of hundred. Given below are the summaries of test results and they were presented with coded lettered **A-F** as indicated in the marking guide above. Mean marks were given for each guide item significance tests at 10% level of significance between the differences in average marks obtained in test 1 and test 2. Finally a significance test was done to test the difference between overall form three mathematics results and the mean of the results in test 1 and test 2.

Table 2d: Summary of tests and form three mathematics results mean marks

Test	**A**	**B**	**C**	**D**	**E**	**F**
Form Three Mathematics Results (Y_3)	-	-	-	-	-	49%
Test 1 (Y_1)	$\frac{8,8}{16}$	$\frac{6.5}{16}$	$\frac{8.1}{16}$	$\frac{8.2}{16}$	$\frac{8.8}{16}$	51%
Test 2 (Y_2)	$\frac{15}{20}$	$\frac{10}{20}$	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{13}{20}$	60%
Total standing Test 1 & 2	$\mathbf{\frac{23.8}{36}}$	$\mathbf{\frac{16.5}{36}}$	$\mathbf{\frac{18.1}{36}}$	$\mathbf{\frac{20.2}{36}}$	$\mathbf{\frac{21.8}{36}}$	**56%**

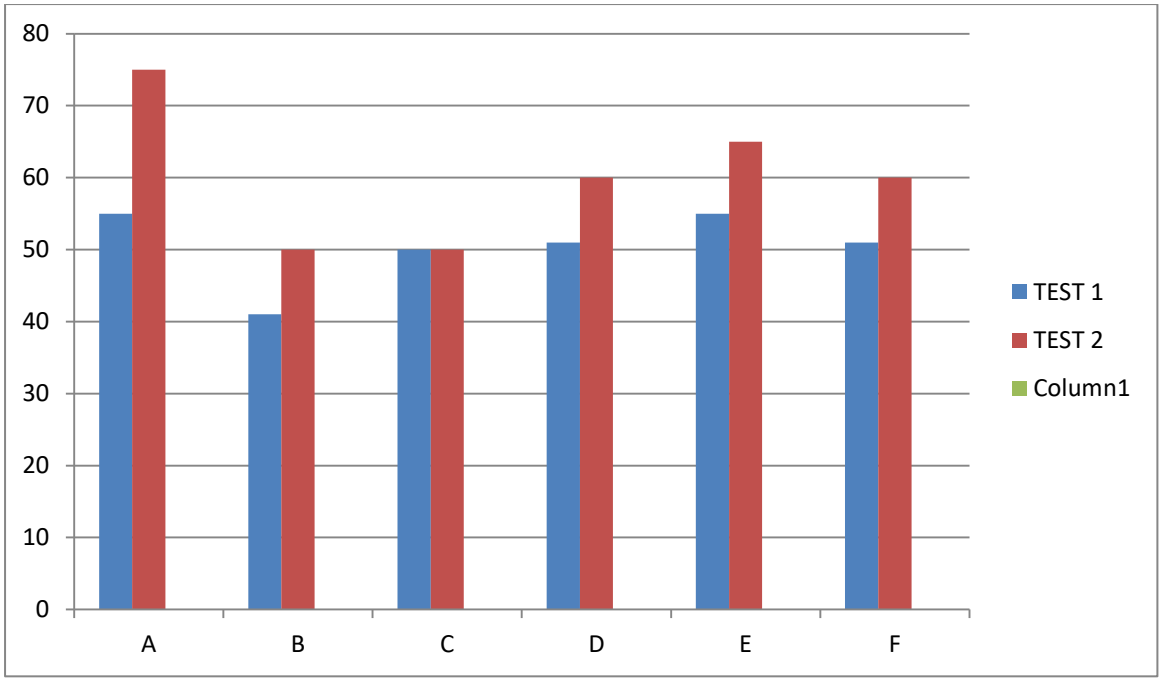

Figure 2: Bar graph comparing test 1 against test 2 item scores

From the bar graph it can be clearly noted that pupils performed better if test items were given using diagrammatic representations on which questions were based. This meant learners understood diagrammatic instructional representations better than worded instructions on mathematical problems. However, statistical tests were carried to test whether there was a significant difference in average performance of learners on tests which used more diagrams from tests with word instructions.

Hypothesis

H_0 : The average performance of learners in solving problems posed in with diagrams **is not higher** than solving problems posed without diagrams.

H_a : The average performance of learners in solving problems posed with diagrams **is higher** from solving problems posed without diagrams.

Level of significance test carried out at 10% level of significance

Test Statistic: Data from computations

$\bar{Y}_1 = 51 \quad \bar{Y}_2 = 60 \qquad S_1=25.8 \quad S_2=22.6 \quad \sigma= 24.8 \quad n_1=n_2=25$

$Z_{calc} = (\bar{Y}_2 - \bar{Y}_1)/\sigma\sqrt{\frac{2}{\square}}$

$Z_{calc}= 1.283$

Rejection Criteria one tailed test with 10% significance level

$Z_{crit} = Z_{0.10} =1.282$ upper tail test.

Reject H_0 if $Z_{calc} > Z_{crit}$ 1.283> 1.282

Decision

Since $Z_{calc} > Z_{crit}$ there is sufficient evidence at 10% level to reject H_0 and a conclusion can be reached that diagrams play a major role in improving performance of learners in problem solving in mathematics. The significance test is a statistical test which is used in inferential statistics to set the validity level on which results from sample data are valid. In case demonstrated above there is probability of less than 10% that diagrams do not play a major role in problem solving in mathematics. Therefore, it can be said from this test that in mathematics problem solving problems which are posed basing the question on given diagrams yield better results than problems presented with less diagrams.

Average percentage scores for test 1, test 2 and form three results

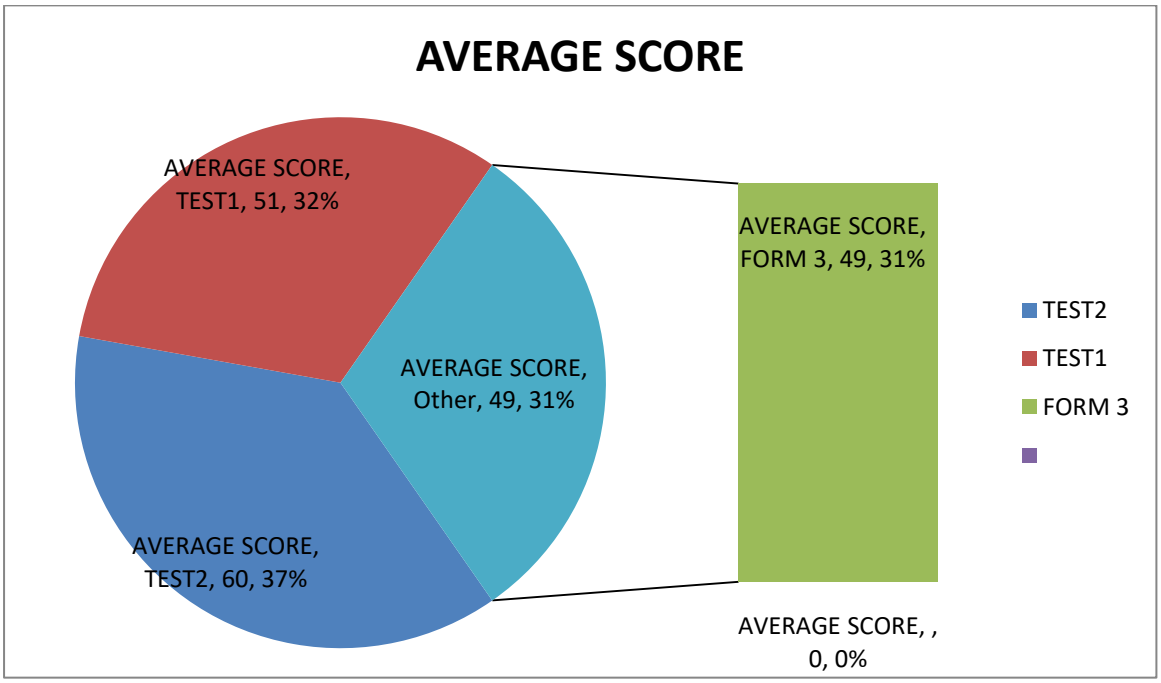

Figure 3: Mean percentage scores for test 1, test 2 and form 3 mathematics results

It can be concluded from this research that the role of diagrams in problem solving in mathematics is most evident in helping learners in understanding the problem. This is based on the test results of tests administered in which one was based on word instructions and the other was based on diagrammatical presentations. Learners at form four level found it difficult to come up with diagrammatical or graphical presentation implied in a problem. Their problem solving abilities rose by almost 10% if problems are posed with diagrams. To answer the two major problem questions was concluded from this study that the role of diagrams in problems solving in mathematics at form four levels was to help learners to understand the problem. Pupils had some difficulties in devising diagrammatic plans in solving mathematical problems.

4.3. DATA DISCUSSION AND INTERPRETATION

Data discussion and interpretation section was organised according to the research questions as follows:

4.3.1 What is the role of diagrams in helping learners show understanding of mathematical problems?

The data presented in the preceding section on this research question reflected that diagrams helped learners overcome linguistic challenges which learners may face in understanding the problem. Understanding the problem was the key to opening avenues for solving mathematical problems, which is step 1 in Polya (1957)'s four step model in problem solving. It was very important to understand the problem before trying to find the solution to it. The findings established the role played by diagrams in helping learners to show understanding of mathematical problems. These findings concurred with earlier findings by Pantziara et al (2004) in a study done in Cyprus that looked on the role of diagrams in solving non-routine problems in mathematics. Although their focus was on non-routine problems, this study added onto their study in that it established the role of diagrams in solving mathematical problems in general. It was noted that the role of diagrams in word problems differed among learners of different abilities. Thus, supporting the results of this study, learners need to be helped to understand the problem before trying to find a solution to it. After understanding the problem, learners then devise and design plans and strategies which is discussed in research question 2.

4.3.2 What is the role of diagrams in helping learners to devise and design plans for solving mathematical problems?

The study revealed that learners should be helped to devise and design plans and strategies for solving mathematical problems after understanding the problem. This corroborated with the second step on George Polya (1957)'s four step model of problem solving which is,

devise and design plans. This study revealed that diagrams provided a pictorial reference which gave a holistic view of the problem. It was noted that the teaching method employed has a bearing in helping learners devise and design strategies in solving mathematical problems. These findings concurred with earlier findings by Elia and Philippou (2004) in a study carried out i Cyprus that revealed that pictures serve representational, informational and organisational functions in solving mathematical problems. This study ascertained the role of diagrams in helping learners devise and design plans in solving mathematical problems through the use of child centred approaches. Learners should be left to

4.3.3 What is the role of diagrams in helping learners implementing devised plans and strategies in solving mathematical problems?

The study brought out that lesson achievement is highly realised when diagrams are used in the teaching and learning episodes. It was shown that diagrams filled the gap which words may fail to clearly state, thus breaking the linguistic barrier. The findings showed that diagrams can be successfully used to solve mathematical problems, hence play a pivotal role the successful teaching and learning. According to Polya (1957), this is step number 3 which is the implementation phase of devised and designed plans. This corroborated with the work done by Pantziara et al (2004) in Cyprus on the role of diagrams in solving non-routine problems. Two tests were used in the study, test A was open and test B was diagrammatic in nature. It was revealed that diagrams assist learners in solving problems. This study showed that diagrams play a pivotal role in helping learners implementing devised and designed plans in solving mathematical problems.

4.3.4 What is the role of diagrams in helping learners to review, reflect and verify solutions obtained in solving problems?

The study reflected that learners performed better in test 2 which was diagrammatic in nature. This clearly outlined the role of diagrams in helping learners review, reflect and verify solutions obtained in solving mathematical problems. This concurred with Poch, van Garderen and Scheuermann (2015)'s study in the United States of America. Although they used learners with learning disabilities, their study outlined the five aspects in the role of diagrams which are; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. This was not divorced to Polya (1957)'s four step model in solving problems. The fourth step which is look back is the reviewing, reflection and verification of solutions obtained in solving mathematical problems. This study built onto Poch et al (2015)'s study in that it outlined the role of diagrams in problem solving in general using mixed ability learners. The statistical tests done outlined that the role of diagrams in problem solving in mathematics is pivotal. Therefore, diagrams provided the means by which learners can communicate their solutions.

4.4. CHAPTER SUMMARY

The chapter dealt with the research findings, data presentation and data analysis. The first part presented biographic data on the learners who participated in the study. Research findings from teacher questionnaires were presented after the biographical data of the learners. Contributions and teachers' feelings and suggestions were also included together with questionnaire findings. Tabular and graphical presentations were provided for the numerical findings. The last part dealt with data presentation and analysis from test 1, test 2 and form three results. A significance test was carried in the last part to statistically establish the probability to which the findings can be representative of the whole population and this was established at 10% level using a Z-test. A summary of the whole study and recommendations were given in the next chapter.

CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 INTRODUCTION

This final chapter of the study gave a brief summary of the study in line with its focus to study objectives or basically the goals of the study. The study was made up of three sections which mainly focused on the objectives of the study and on how far the study had been focused on the study questions. Finally the researcher gave recommendations in line with the research findings.

5.1 SUMMARY

The purpose of the study was to establish the role of diagrams in problem solving in mathematics at form four level. The research was carried out at Chifedza High School in Chivi District. The subjects of the study were sampled from teachers and form four formal and non-formal learners from Chifedza High School. Ten teachers from the Mathematics department and Science department were purposively called upon to respond to teacher questionnaire on how they place the role of diagrams in solving mathematical problems. A stratified random sample of thirty form four students of normally distributed abilities was selected to respond to a questionnaire and to take on two types of tests. The marking guides of the tests focused on the objectives of the study and marks were awarded accordingly.

It was costly to print questionnaires and test items especially in these harsh economic conditions. However, an income generating project started by the researcher assisted him immensely in carrying out this study. Some of the respondents took too long to complete the questionnaires and follow up was done since they were convenient to the researcher.

The study established that the success of learners in problem solving in mathematics is strongly dependent on how learners understand the problem situation. Learners failed to establish clear steps in solving mathematical problems when worded instructions on diagrams

are given, in test 1, but they fared better in cases where diagrams were widely used, in test 2. Data analysis was strongly based on the numerical tabulated and graphical data from which comments and conclusions were drawn in line with the goal of the study. A 10% z-test was carried to test the level of significance of the conclusions drawn from the study. The results from the z-test showed that the results can be trusted to over 90% probability of reliability to be representative of the role of diagrams in problem solving in mathematics. The major findings of the study reveal that diagrams play a very important role in problem solving in mathematics.

5.2 CONCLUSION

From questionnaires it was discovered that teachers use traditional teaching methods although they are aware that learner centred techniques are meaningful to learners than the rote and lecture methods. Mathematics teachers and Science teachers who responded to questionnaires pointed out that:

i. Diagrams fill in the gap which words may fail to clearly state
ii. Diagrams provide the means by which learners can communicate their solutions.
iii. Diagrams provide a pictorial reference which gives a holistic view of the problem.
iv. Diagrams help learners to overcome linguistic challenges which learners may face in understanding the problem.

From the above views it can be noted that the major role of diagrams in problem solving in mathematics is to help learners to;

- understand the problem
- apply mathematical formulae and theories
- display their solutions confidently

- provide visual and concrete means for revising and reviewing their solutions
- overcome linguistic barriers that may arise in interpreting the problem or in providing a solution to a problem

These showed that diagrams play a major role in helping learners understand mathematical problems as objective A ranked higher both in test 1 and test 2. In test 1 an average of 55% and test 2 an average of 75%.

Comparing test 1 and test 2 results was carried out using a significance test and it was confirmed that diagrams help learners understand mathematical problems much better if test items are asked basing on diagrams.

5.3 RECOMMENDATIONS

From the findings which were not so many in this case but many recommendations can be suggested. However, the writer strongly recommends that, since diagrams help learners to understand problems more clearly then, mathematical problems should be posed using diagrams to enhance correct and clear understanding of the problem. The writer also recommends that any form of instruction on mathematical problem should accompany diagrams and it is hoped that this provides a strong starting point in the sharpening of problem solving skills. The researcher also noted that teachers should be urged to use child centered approaches with a variety of instructional media which arouse interest and enhance understanding of mathematical problems when teaching. There is need for teachers to vary their teaching methods and strategies depending on the topic to be taught. This is important in clarifying the various concepts in different topics.

The researcher also recommends strongly that parents should strive to know the performance of their students. Obama (2004) argues that parents have the primary responsibility of

instilling the value of hard work and educational achievement in their children. Parents should work hand in glove with teachers so that pupils will receive both guidance and encouragement from both parties. On the other hand, the researcher recommends the Ministry of Primary and Secondary Education to avail graphical, diagrammatical and visual-audio media for schools which would help learners to develop and hone their problem solving capabilities. The school should provide students with the requisite resources to enhance their academic potential by improving their problem solving techniques. Furthermore, the researcher recommends that it is our duty to iron out the language barrier in communicating problems by expressing mathematical problems extensively using diagrammatic, graphical and pictorial representations. This enhances clear understanding and expression of the problem situation.

Finally the writer recommends that further studies and research should be carried out in this area to refine this crude attempt for the better and obtain much more useful results.

Let this study be one little step towards further studies in this area and not fall to the usual fate of being shelved to gather dust.

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APPENDIX A: QUESTIONNAIRE FOR TEACHERS

Questionnaire for teachers

The researcher is an undergraduate student with Midlands State University currently doing Bachelor of Education Degree in Mathematics. The topic under study is entitled " An investigation into the Role of Diagrams in Problem Solving in Mathematics at Form 4 Level at Chifedza High school". Information gathered will assist teachers, learners and other stakeholders to decide more effective strategies in teaching and learning Mathematics to overcome the challenges faced in the subject. Your cooperation in filling in the questionnaire is highly valued for the success of the study. Be assured that all information you supply will be treated with utmost confidentiality. You are entitled to fill both section A and B of the questionnaire

Place a tick in an appropriate box where necessary ☐

Section A: Demographic Information

1) Gender

Male ☐ Female ☐

2) Marital Status

Single ☐ Married ☐

3) Age

Below 25 ☐

26-30 ☐

31-35 ☐

36 -40 ☐

41 and above ☐

4) Highest academic qualification

'O' level ☐

'A' level ☐

B.Sc. ☐

Other ☐ Specify----------

5) Highest professional qualification

☐

Grad. C. Ed.

PGDE ☐

C. Edu ☐

Dip in Ed ☐

B. Ed ☐

Other ☐ Specify----------

6) Teaching experience in years at Ordinary Level

0-5 ☐

6-10 ☐

11-15 ☐

16-20 ☐

21-25 ☐

26 and above ☐

Section B: Teaching methods and Diagrammatic competence

1) Do you think Teaching methods and use of diagrams employed by the teacher affect students' performance in Problem Solving?

Yes ☐ No ☐

2) Have you found use of diagrams useful in your teaching?

Yes ☐ No ☐

3) Do you think diagrams reduce the labour of solving problems?

Yes ☐ No ☐

4) Do you advise your learners to sketch diagrams before solving problems?

Yes ☐ No ☐

5) Which textbooks do you use most often in teaching mathematics?

New General Mathematics ☐ Focus on Maths ☐ Maths Today ☐ Other ☐
specify other ……

Section C: Teaching Methods and Diagrammatic Competence

1) How often do you use diagrams and/or graphical in your lessons per week?

---

2) Which teaching method do you use in your lessons?

Rating of the method preferred is as follows:

N-never, S-sometimes, O- often, VO- very often, and A- always

Teaching Method	N	S	O	VO	A
Lecture					
Problem solving					
Small group discussion					
Pupil presentation					

3) Can you state your own feelings on the role of diagrams in problem solving in mathematics?

---

4) How do you rate your lessons in which diagrammatic and graphical presentation feature most in terms of learners' participation and understanding?

Excellent ☐ Fair ☐ Poor ☐

Comment on your respond above

Section D: Diagrams and Problem Solving Approach

1) Are you conversant with the Problem Solving approach as a strategy in teaching Mathematics? State your understanding of the approach.

---

2) Generally can you site any merits and drawbacks in the use of diagrams in the Problem solving approach in mathematics?

MERITS----------

DRAWBACKS----------

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3) Is Problem Solving Approach applicable in the teaching and learning of Mathematics in Zimbabwe and
why?.....
.....
.....
.....
.....
.....
.....

Thank you for taking the time to complete this questionnaire.

APPENDIX B : QUESTIONNAIRE FOR LEARNERS

Questionnaire for Learners

The purpose of this questionnaire is to get a better understanding on the Role of Diagrams in Problem Solving in Mathematics at Form 4 Level. Your answers and comments will be used for the study it intends and not shared with anyone else. Your honest and complete response will be greatly appreciated.

Section A

Student's characteristics

1) Gender

Male ☐ Female ☐

2) Age

15-17 ☐ 18-20 ☐ Above 20 ☐

3) Are you a:

Day student ☐ Boarder student ☐ Non-formal student ☐

4) Mark scored in Mathematics in third term at Form 3 level.

☐

Section B: Students Attitudes towards role of diagrams in problem solving in mathematics.

1) Do you face difficulties in solving mathematical problems using diagrams?

Yes ☐ No ☐

2) If **Yes** which concepts or topics are challenging to you?

---

3) How do you rate your performance in mathematics?

Excellent ☐ Good ☐ Fair ☐ Poor ☐

4) How do you think diagrams affect your performance in mathematics?

☐

Reduce difficulties ☐ increase difficulties

5) Give a brief explanation of your answer above.

---

6) At which stage or part do you think diagrams should be used most in the teaching and learning of mathematics?

ANSWERING TEST ITEMS ☐ TEST AND EXAM ITEMS ☐

ON BOTH ☐

7) Are diagrams often used in your maths lessons?

Yes ☐ No ☐

8) Does technological advancement like internet assist you in learning Mathematics?

Yes ☐ No ☐

9) Do you have adequate textbooks with diagrammatic and graphical presentations at your school?

Yes ☐ No ☐

10) Which text book do you use at your school from those given below

New General Mathematics ☐ Focus on Maths ☐ Maths Today ☐

Other ☐ specify other…….

Thank you for your cooperation.

APPENDIX C

TEST ITEMS 1

1) Find the area and perimeter of the shapes described below.

(a) A kite ABCD with AB = AD = 8cm and BC = DC = 15cm and ABC = ADC = 90^0 .

(b) A rhombus PQRS with PQ = 6m and PQR = 120^0.

(c) An equilateral triangle XYZ with XY = 12cm.

2) (a) Farmer Joe wishes to erect a barrier of a straight fence 75m long. He wishes to place standard posts 15m apart. How many standard posts does he need?

(b) Farmer Jerry also wishes to fence off his rectangular plot measuring 100m by 75m using four strands of barbed wire and leaving a 5m space for the gate at one of the corners. He also plans to place standard posts 15m apart.

(i) If he used five corner posts for the corners and gate space how many standard posts does he have to buy?

(ii) What is the total length of barbed wire he has to such that he can fence his plot excluding the gate space and also uses 2m for tying and securing?

3. X, Y and Z are 3 towns such that the bearing of town Z from town X is 060. Town Y is due east of town X and town Z is due north of town Y.

a) State the special name given to the triangle formed by the location of the 3 towns XYZ.

b) Calculate angle formed at town X.

c) Find i) the 3 figure bearing of town X from town Z.

ii) the compass bearing of town X from town Z.

4. A composite solid in the form of a hut is made up of a cylinder and a cone with a common diameter of 6 meters. The slant height of the cone is 5 meters and the height of the cylinder is 5 meters. Take π to be 3,142. Find;

a) the height of the cone.

b) the curved surface area of the cylinder.

c) the volume of the composite solid.

5. Triangle A has vertices at (-3;1), (-2;1) and (-2;3). Triangle B is the image of triangle A under a reflection with the y-axis being the mirror line.

a) Write down the coordinates of triangle B.

b) Triangle C with vertices (0;-1), (1;-1) and (1;1) is the image of triangle A under a single transformation. Describe the single transformation fully.

6. A, B and C are 3 villages on level ground such that the distance between village A and village B is 5 kilometers, village A and village C is 10m and angle at village A is 30 degrees. Calculate;

a) the distance between village B and village C.

b) the angle at village B to the nearest degree.

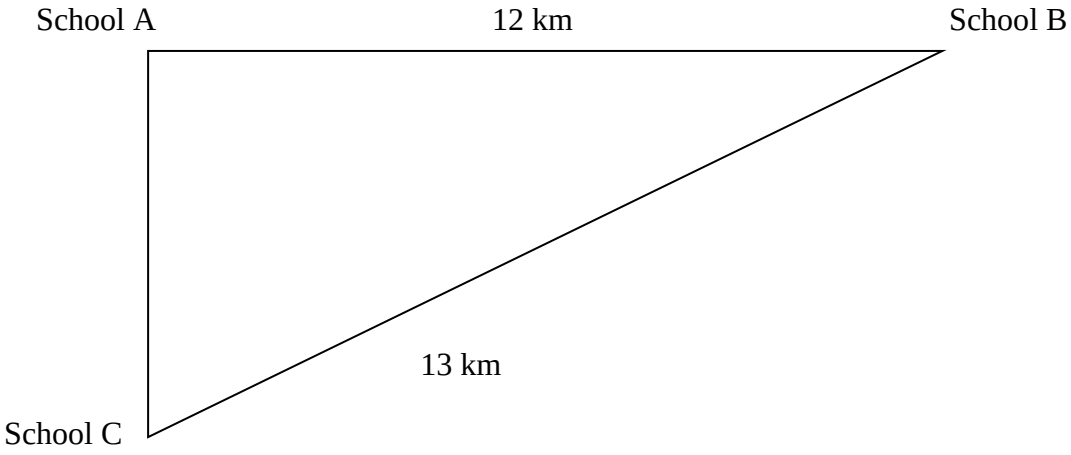
c) the area of the triangle formed by the three villages.

.

APPENDIX D

TEST ITEMS 2

1 In the diagram A, B and C are 3 schools on level ground connected by a tarred road which forms a right angled triangle with a right angle at A. The distance A to B is 12 kilometres and the distance B to C is 13 kilometres.



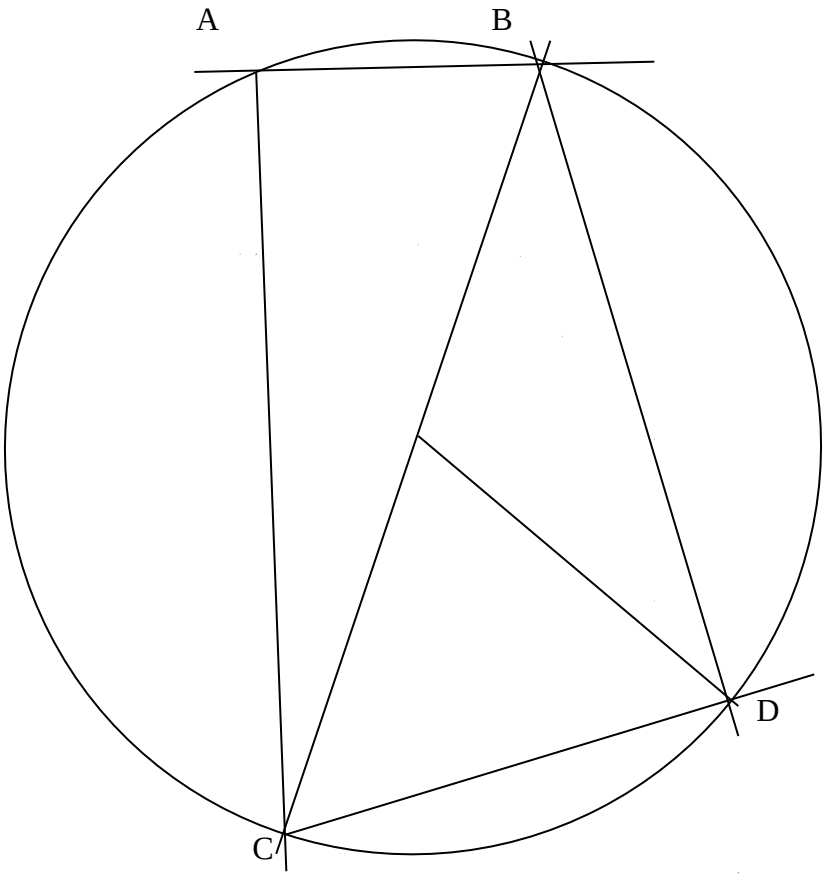
Find

a) the distance between school A and school C,

b) the length of the tarred road,

c) the area surrounded by the tarred road.

2. In the diagram ABCD is a circle centre O. BOC is the diameter to the circle. Angle COD=60°. Angle ACB=30°



a).What is the special name given to quadrilateral ABCD.

b) Find;

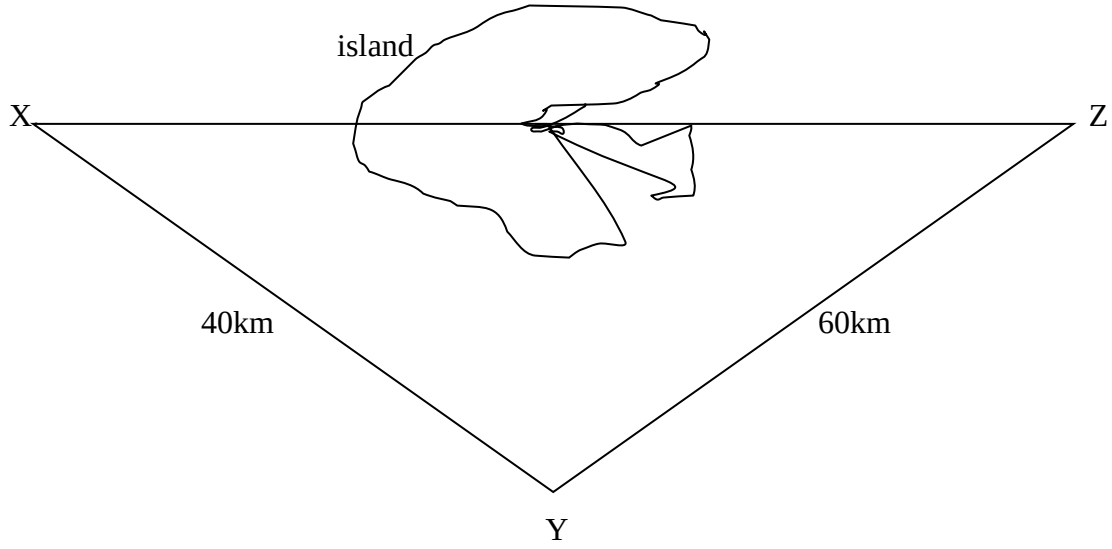
i) angle BAC,

ii) angle ABC,

iii) angle CBD,

iv) angle OCD.

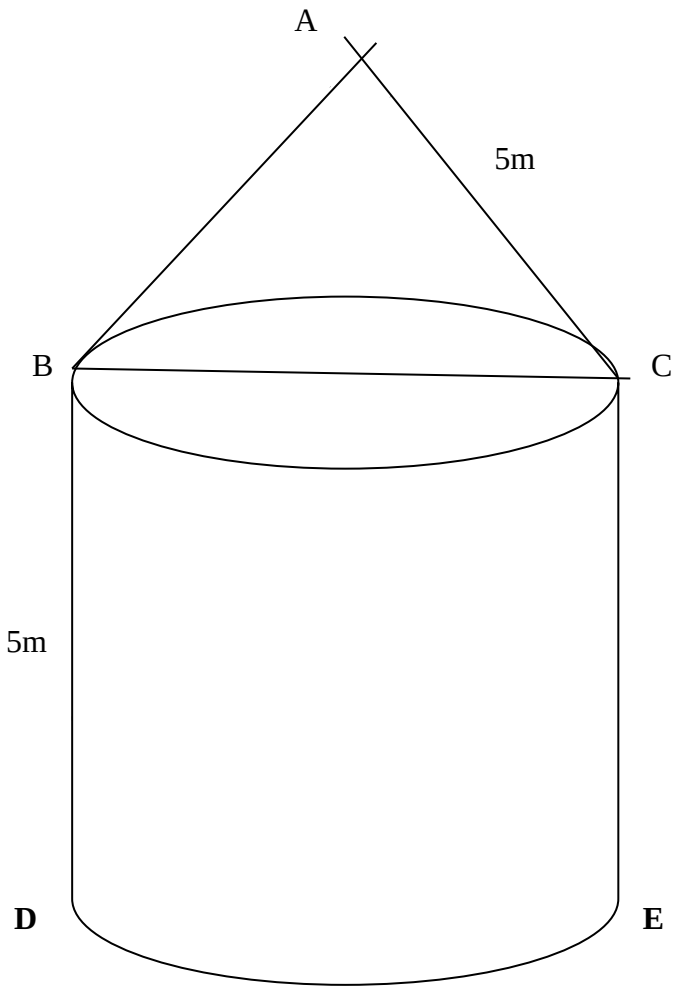
3. To avoid an island, a ship travels 40km from port X to port Y and then 60km from port Y to port Z. The angle formed at port Y is 120 degrees.



Calculate;

a) The distance between port X and port Z,
b) The angle at port X,
c) The shortest distance from port Y to route XZ,
d) Area of the triangle formed by the three ports.

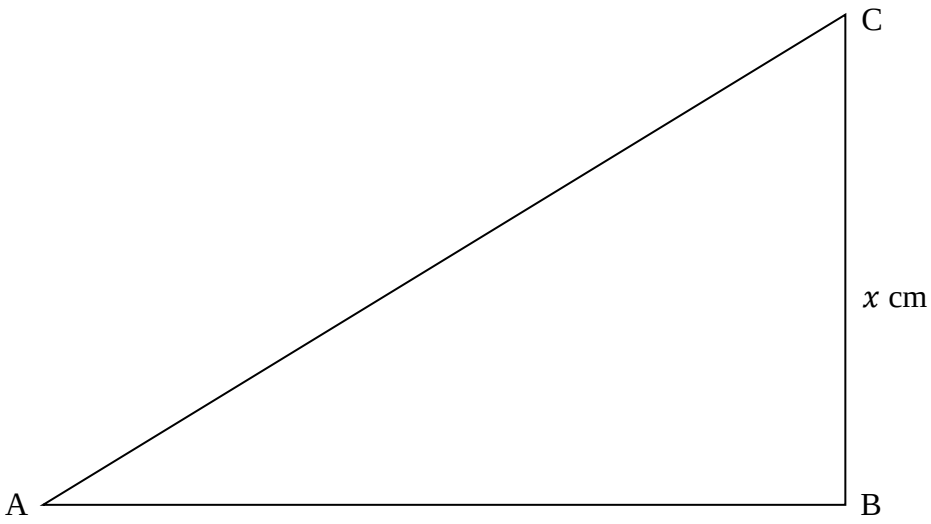
4. In the diagram, ABCDE is a composite solid made up of a cylinder and a cone. The diameter of the cone and cylinder is 6 meters. The slant height of the cone is 5 meters and the height of the cylinder is 5 meters. In this question take π to be 3,142.



Calculate;

a) The height of the cone,
b) The curved surface area of the cylinder,
c) The volume of the composite solid.

5.



a) When the area of triangle ABC is 48 square centimetres,
   i) Show that $x^2 + 4x - 96 = 0$,
   ii) Solve the equation $x^2 + 4x - 96 = 0$.
   iii) Write down the length of AB.

b) When tan y= $\frac{1}{6}$, find the value of x.

c) When the length of AC=9 cm.
   i) Show that $2x^2 + 8x - 65 = 0$.
   ii) Solve the equation $2x^2 + 8x - 65 = 0$, giving your answers correct to 2 decimal places.
   iii) Calculate the perimeter of triangle ABC.

APPENDIX E - CLEARANCE LETTER FROM THE MINISTRY OF PRIMARY AND SECONDARY EDUCATION

All communications should be addressed to "The Secretary for Primary and Secondary Education"
Telephone: 799914 and 705153
Telegraphic address : "EDUCATION"
Fax: 791923

ZIMBABWE

Reference: C/426/Masvingo
Ministry of Primary and Secondary Education
P.O Box CY 121
Causeway
ZIMBABWE

13 October 2017

THE HEAD
CHIFEDZA HIGH SCHOOL
18 OCT 2017
PRIVATE BAG 553, CHIVI
TEL: 0337 - 429

MIN. OF PRIMARY & SECONDARY EDUCATION
HUMAN RESOURCES DEPARTMENT
CHIVI
17 OCT 2017
PRIVATE BAG 646
CHIVI

Magwenzi Godfree
Cfifedza High School
Private Bag 553
Chivi

RE: PERMISSION TO CARRY OUT RESEARCH IN MASVINGO PROVINCE: CHIVI DISTRICT: CHIFEDZA HIGH SCHOOL

Reference is made to your application to carry out a research in the above mentioned schools in Masvingo Province on the research title:

"AN INVESTIGATION INTO THE ROLE OF DIAGRAMS IN PROBLEM SOLVING IN MATHEMATICS AT FORM 4 LEVEL AT CHIFEDZA HIGH SCHOOL IN CHIVI DISTRICT"

Permission is hereby granted. However, you are required to liaise with the Provincial Education Director Masvingo Province, who is responsible for the schools which you want to involve in your research. You should ensure that your research work does not disrupt the normal operations of the school. Where student are involved, parental consent is required

You are required to provide a copy of your final report to the Secretary for Primary and Secondary Education.

MIN. OF PRIMARY & SECONDARY EDUCATION
POLICY PLANNING, RESEARCH AND STATISTICS
13 OCT 2017
P.O. BOX CY 121, CAUSEWAY

E.Chinyowa
Acting Director: Planning, Research and Statistics
For: **SECRETARY FOR PRIMARY AND SECONDARY EDUCATION**
cc: PED Masvingo Province

ALL communications should be addressed to
"The Provincial Education Director for Primary and Secondary Education"
Telephone: 263585/264331
Fax: 039-263261

ZIMBABWE

Ministry of Primary and Secondary Education
P. O Box 89
Masvingo

17 October 2017

MIN. OF PRIMARY & SECONDARY EDUCATION
HUMAN RESOURCES DEPARTMENT
CHIVI
17 OCT 2017
PRIVATE BAG 545
CHIVI
TEL : 0337 497 / 278 / 525

Magwenzi Godfree
Chifeza High School
Private Bag 553
Chivi

RE: PERMISSION TO CARRY OUT RESEARCH IN MASVINGO PROVINCE: CHIVI DISTRICT: CHIFEZA HIGH SCHOOL.

Reference is made to your application to carry out a research at the above mentioned school in Chivi District on the research title:

"AN INVESTIGATION INTO THE ROLE OF DIAGRAMS IN PROBLEM SOLVING IN MATHEMATICS AT FORM 4 LEVEL AT CHIFEZA HIGH SCHOOL IN CHIVI DISTRICT."

Please be advised that the Secretary for Primary and Secondary Education has granted permission to carry out your research.

You are also advised to liaise with the District Education Officer who is responsible for the schools which are part of the sample for your research.

MINISTRY OF PRIMARY & SECONDARY EDUCATION
PROVINCIAL EDUCATION DIRECTOR
MASVINGO
17 OCT 2017
P.O. BOX 89, MASVINGO
ZIMBABWE
TEL : 039 - 264331
FAX : 039 - 263261

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CHIFEDZA HIGH SCHOOL
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Z. M. Chitiga
Provincial Education Director
MASVINGO PROVINCE